Test Prep 4
Answer key

KEY

1. Find $\cos(\theta)$ given θ is in Quadrant 1. Enter your answer in simplified radical form.

$\sin(\theta) = \frac{4}{7}$

$\cos\theta = \frac{\sqrt{33}}{7}$

2. Find $\csc(\theta)$ given θ is in Quadrant 2. Enter your answer in simplified radical form.

$\cot(\theta) = \frac{4}{3}$

$\csc\theta = \frac{5}{3}$

3. Find $\sin(\theta)$ given θ is in Quadrant 3. Enter your answer in simplified radical form.

$\cos(\theta) = \frac{5}{6}$

$\sin\theta = -\frac{\sqrt{11}}{6}$

4. Find $\sec(\theta)$ given θ is in Quadrant 4. Enter your answer in simplified radical form.

$\tan(\theta) = \frac{7}{8}$

$\sec\theta = \frac{\sqrt{113}}{8}$

5. Find c if $2c \cdot \tan(150^{\circ}) = 6$

$-3\sqrt{3}$

6. Solve for x, with $0 < x < \pi$. Write the answer in radians.

$-6\cot(x) + 8 = -\cot(x) + 13$

$\frac{3\pi}{4}$

7. Solve for x, with $-\frac{\pi}{2} < x < \frac{\pi}{2}$. Write the answer in radians.

$3\tan(x) + 5\sqrt{3} = \tan(x) + 3\sqrt{3}$

$-\frac{\pi}{3}$

8. Solve for x, with $0 < x < \pi$. Write the answer in radians.

$7\cos(x) + 4\sqrt{3} = \cos(x) + 7\sqrt{3}$

$\frac{\pi}{6}$

9. Solve for x, with $-\frac{\pi}{2} < x < \frac{\pi}{2}$. Write the answer in radians.

$\sin(x) + \frac{1}{2} = -\sin(x) + \frac{3}{2}$

$\frac{\pi}{6}$

10. Solve for x, with $-\frac{\pi}{2} < x < \frac{\pi}{2}$. Write the answer in radians.

$-2\csc(x) + \sqrt{2} = -\csc(x) + 2\sqrt{2}$

$-\frac{\pi}{4}$

11. Solve for x, with $0 < x < \pi$. Write the answer in radians.

$-3\sec(x) - \frac{4\sqrt{3}}{3} = -2\sec(x) - \frac{2\sqrt{3}}{3}$

$\frac{5\pi}{6}$

12. Find the value of the expression below using the sum and difference formulas.

$\cos\left(\frac{\pi}{6} + \frac{\pi}{4}\right)$

$\frac{\sqrt{6}-\sqrt{2}}{4}$

13. Find the value of the expression below using the sum and difference formulas.

$\cos\left(\frac{7\pi}{4} - \frac{\pi}{3}\right)$

$\frac{\sqrt{2}-\sqrt{6}}{4}$

14. Use sum and difference formulas to write the expression below as a trigonometric function of one angle. e.g. $\sin(30^\circ)$. Do not evaluate.

$\cos(56^\circ)\cos(71^\circ) + \sin(56^\circ)\sin(71^\circ)$

$\cos(15)$

15. Find the value of the expression below using the sum and difference formulas.

$\sin\left(\frac{\pi}{4} + \frac{4\pi}{3}\right)$

$\frac{-\sqrt{2}-\sqrt{6}}{4}$

16. Find the value of the expression below using the sum and difference formulas.

$\sin\left(\frac{4\pi}{3} - \frac{5\pi}{4}\right)$

$\frac{\sqrt{6}-\sqrt{2}}{4}$

17. Use sum and difference formulas to write the expression below as a trigonometric function of one angle. e.g. $\sin(30^\circ)$. Do not evaluate.

$\sin(57^\circ)\cos(67^\circ) - \sin(67^\circ)\cos(57^\circ)$

$\sin(-10)$

18. Find the value of the expression below using the sum and difference formulas.

$\tan\left(\frac{\pi}{4} + \frac{\pi}{3}\right)$

$-2 - \sqrt{3}$

19. Find the value of the expression below using the sum and difference formulas.

$\tan\left(\frac{7\pi}{6} - \frac{\pi}{4}\right)$

$-2 + \sqrt{3}$

20. Use sum and difference formulas to write the expression below as a trigonometric function of one angle. e.g. $\tan(30^\circ)$. Do not evaluate.

$$\frac{\tan(74^\circ) + \tan(64^\circ)}{1 - \tan(74^\circ)\tan(64^\circ)}$$

$\tan(138)$

21. Find $\sin 2\theta$ given the information below.

Given that $\tan\theta = -\frac{3}{6}$ and θ is in quadrant IV, find $\sin 2\theta$.

$-\frac{36}{45}$

22. Find $\cos 2\theta$ given the information below.

Given that $\tan\theta = \frac{4}{5}$ and θ is in quadrant I, find $\cos 2\theta$.

$\frac{9}{41}$

23. Find $\tan 2\theta$ given the information below.

Given that $\tan\theta = \frac{8}{3}$ and θ is in quadrant III, find $\tan 2\theta$.

$-\frac{48}{55}$

24. Apply the half angle identities to evaluate the expression.

Evaluate $\sin(105^\circ)$ using a half-angle identity.

$\sqrt{\left(\frac{1}{2} + \frac{\sqrt{3}}{4}\right)}$

25. Apply the half angle identities to evaluate the expression.

Evaluate $\cos\left(22.5^{\circ}\right)$ using a half-angle identity.

$\sqrt{\left(\frac{1}{2}+\frac{\sqrt{2}}{4}\right)}$

26. Apply the half angle identities to evaluate the expression.

Evaluate $\tan\left(67.5^{\circ}\right)$ using a half-angle identity.

$\frac{\left(\frac{\sqrt{2}}{2}+\frac{1}{2}\right)}{\left(\frac{1}{2}\right)}$

27. Consider the angle θ in the first quadrant. Given that $0^{\circ}<\theta<360^{\circ}$ and $\tan\theta=\frac{7}{4}$, find the exact value of $\sin\frac{\theta}{2}$.

$\sqrt{\frac{1}{2}-\frac{2\sqrt{65}}{65}}$

28. Consider the angle θ in the fourth quadrant. Given that $0^{\circ}<\theta<360^{\circ}$ and $\tan\theta=-\frac{3}{7}$, find the exact value of $\cos\frac{\theta}{2}$.

$-\sqrt{\frac{1}{2}+\frac{7\sqrt{58}}{116}}$

29. Consider the angle θ in the second quadrant. Given that $\tan\theta=-\frac{5}{6}$, find the exact value of $\tan\frac{\theta}{2}$. Rationalize any denominators.

$\frac{\sqrt{61}+6}{5}$

30. Evaluate the product using a sum or difference of two functions.

$\cos\left(210^{\circ}\right)\cos\left(30^{\circ}\right)$

$-\frac{3}{4}$

31. Evaluate the product using a sum or difference of two functions.

$\sin\left(120^{\circ}\right)\sin\left(30^{\circ}\right)$

$\frac{\sqrt{3}}{4}$

32. Evaluate the product using a sum or difference of two functions.

$\sin\left(300^{\circ}\right)\cos\left(30^{\circ}\right)$

$-\frac{3}{4}$

33. Evaluate the sum by converting it to a product of two functions.

$\sin\left(210^{\circ}\right)+\sin\left(30^{\circ}\right)$

0

34. Evaluate the difference by converting the expression to a product of two functions.

$\sin\left(330^{\circ}\right)-\sin\left(30^{\circ}\right)$

-1

35. Evaluate the difference by converting the expression to a product of two functions.

$\cos\left(150^{\circ}\right)-\cos\left(30^{\circ}\right)$

$-\sqrt{3}$

36. Rewrite the expression below as a product.

$\cos\left(210^{\circ}\right)+\cos\left(30^{\circ}\right)$

0

37. Solve the triangle. Assume α is opposite side a, β is opposite side b, and γ is opposite side c. Enter each angle measure in degrees. Round sides and angles to the tenths place if necessary.

$\alpha = 55^\circ$
$\gamma = 46^\circ$
$a = 17$

$\beta = 79$
$b = 20.4$
$c = 14.9$

38. Solve the triangle. Assume α is opposite side a, β is opposite side b, and γ is opposite side c. Enter each angle measure in degrees. Round sides and angles to the tenths place if necessary.

$\beta = 42^\circ$
$c = 14$
$b = 14$

$\gamma = 42$
$\alpha = 96$
$a = 20.8$

39. Solve the triangle. Assume α is opposite side a, β is opposite side b, and γ is opposite side c. Enter each angle measure in degrees. Round sides and angles to the tenths place if necessary.

$\beta = 64^\circ$
$c = 16$
$a = 11$

$\alpha = 41.5$
$\gamma = 74.5$
$b = 14.9$

40. Solve the triangle. Assume α is opposite side a, β is opposite side b, and γ is opposite side c. Enter each angle measure in degrees. Round to the tenths place.

$a = 20$
$c = 14$
$b = 21$

$\alpha = 66.2$
$\beta = 73.9$
$\gamma = 39.8$

$$\frac{1+\sec^2\theta}{\sec^2\theta} = 1+\cos^2\theta$$

$$\frac{1}{\sec^2\theta} + \frac{\sec^2\theta}{\sec^2\theta} = 1+\cos^2\theta$$

$$\cos^2\theta + 1 = 1+\cos^2\theta$$

$$\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta} = \frac{1}{\cos\theta\sin\theta}$$

$$\frac{\sin\theta(\sin\theta)+\cos\theta(\cos\theta)}{\sin\theta\cos\theta} = \frac{1}{\cos\theta\sin\theta}$$

$$\frac{\sin^2\theta+\cos^2\theta}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$$

$$\frac{1}{\sin\theta\cos\theta} = \frac{1}{\sin\theta\cos\theta}$$

$$\sec^2\theta - \sin^2\theta\sec^2\theta = 1$$

$$\sec^2\theta(1-\sin^2\theta)=1$$

$$\sec^2\theta(\cos^2\theta)=1$$

$$\frac{1}{\cos^2\theta}(\cos^2\theta)=1$$

$$\frac{\cos^2\theta}{\cos^2\theta}=1 \quad 1=1$$

	$\sin\theta$	-1
$\sin\theta$	$\sin^2\theta$	$-\sin\theta$
-1	$-\sin\theta$	$+1$

	$-\sin\theta$	$+1$
$\sin\theta$	$\cancel{-\sin^2\theta}$	$\cancel{\sin\theta}$
$+1$	$\cancel{-\sin\theta}$	1

$$\frac{\{\sin^2\theta-2\sin\theta+1\}}{\sin\theta-1} = \sin\theta-1 \leftarrow \sin\theta - 1$$

$$\frac{(\sin\theta-1)\cancel{(\sin\theta-1)}}{\cancel{\sin\theta-1}} = \sin\theta-1$$

$$\sin\theta-1=\sin\theta-1$$

$$\frac{1}{1-\sin\theta} + \frac{1}{1+\sin\theta}$$

$$\frac{1(1+\sin\theta)+1(1-\sin\theta)}{\{(1-\sin\theta)(1+\sin\theta)\}}$$

$$\frac{1+\cancel{\sin\theta}+1-\cancel{\sin\theta}}{1-\sin^2\theta}$$

$$\frac{2}{\cos^2\theta}$$

$$2\sec^2\theta$$

$$\cot\theta - \frac{\csc^2\theta}{\cot\theta}$$

$$\frac{\cot^2\theta}{\cot\theta} - \frac{\csc^2\theta}{\cot\theta}$$

$$\frac{\cot^2\theta-\csc^2\theta}{\cot\theta}$$

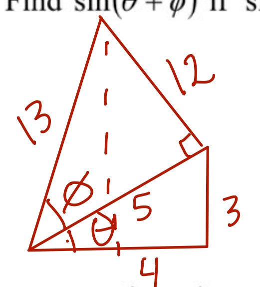
$$\frac{-1}{\cot\theta}$$

$$-\tan\theta$$

Find $\tan(\theta+\phi)$ if $\tan\theta = \frac{1}{3}$ and $\tan\phi = \frac{2}{3}$.

$$\frac{\frac{1}{3}+\frac{2}{3}}{1-\left(\frac{1}{3}\right)\left(\frac{2}{3}\right)} \quad \frac{1}{1-\frac{2}{9}} = \frac{1}{7/9} = \frac{9}{7}$$

Find $\sin(\theta+\phi)$ if $\sin\theta = \frac{3}{5}$, $\sin\phi = \frac{12}{13}$ and both θ and ϕ are quadrant I angles.



$$\sin(\theta+\phi) = \sin\theta\cos\theta + \sin\phi\cos\phi$$

$$\frac{3}{5}\cdot\frac{4}{5} + \frac{12}{13}\cdot\frac{5}{13} = \frac{12}{25} + \frac{60}{169} = \frac{3528}{4225}$$