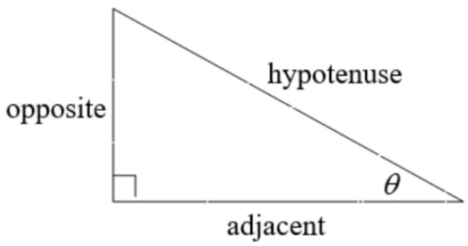
Right triangle definition

For this definition we assume that $0 < \theta < \frac{\pi}{2}$ or $0^\circ < \theta < 90^\circ$.



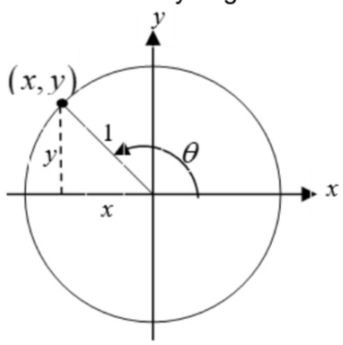
$$\sin(\theta) = \frac{\text{opposite}}{\text{hypotenuse}} \qquad \csc(\theta) = \frac{\text{hypotenuse}}{\text{opposite}}$$

$$\cos(\theta) = \frac{\text{adjacent}}{\text{hypotenuse}} \qquad \sec(\theta) = \frac{\text{hypotenuse}}{\text{adjacent}}$$

$$\tan(\theta) = \frac{\text{opposite}}{\text{adjacent}} \qquad \cot(\theta) = \frac{\text{adjacent}}{\text{opposite}}$$

Unit Circle Definition

For this definition θ is any angle.



$$\sin(\theta) = \frac{y}{1} = y \qquad \csc(\theta) = \frac{1}{y}$$

$$\cos(\theta) = \frac{x}{1} = x \qquad \sec(\theta) = \frac{1}{x}$$

$$\tan(\theta) = \frac{y}{x} \qquad \cot(\theta) = \frac{x}{y}$$

Facts and Properties

Domain

The domain is all the values of θ that can be plugged into the function.

$\sin(\theta)$, θ can be any angle

$\cos(\theta)$, θ can be any angle

$\tan(\theta)$, $\theta \neq \left(n + \frac{1}{2}\right)\pi,\ n = 0, \pm1, \pm2, \ldots$

$\csc(\theta)$, $\theta \neq n\pi,\ n = 0,\ \pm1,\ \pm2, \ldots$

$\sec(\theta)$, $\theta \neq \left(n + \frac{1}{2}\right)\pi,\ n = 0, \pm1, \pm2, \ldots$

$\cot(\theta)$, $\theta \neq n\pi,\ n = 0, \pm1, \pm2, \ldots$

Period

The period of a function is the number, T, such that $f(\theta + T) = f(\theta)$. So, if ω is a fixed number and θ is any angle we have the following periods. $\omega = k$

$$\sin(\omega\theta) \quad \rightarrow \quad T = \frac{2\pi}{\omega}$$

$$\cos(\omega\theta) \quad \rightarrow \quad T = \frac{2\pi}{\omega}$$

$$\tan(\omega\theta) \quad \rightarrow \quad T = \frac{\pi}{\omega}$$

$$\csc(\omega\theta) \quad \rightarrow \quad T = \frac{2\pi}{\omega}$$

$$\sec(\omega\theta) \quad \rightarrow \quad T = \frac{2\pi}{\omega}$$

$$\cot(\omega\theta) \quad \rightarrow \quad T = \frac{\pi}{\omega}$$

Range

The range is all possible values to get out of the function.

$$-1 \le \sin(\theta) \le 1 \qquad -1 \le \cos(\theta) \le 1$$

$$-\infty < \tan(\theta) < \infty \qquad -\infty < \cot(\theta) < \infty$$

$$\sec(\theta) \ge 1 \text{ and } \sec(\theta) \le -1 \qquad \csc(\theta) \ge 1 \text{ and } \csc(\theta) \le -1$$

Definition

$y = \sin^{-1}(x)$ is equivalent to $x = \sin(y)$

$y = \cos^{-1}(x)$ is equivalent to $x = \cos(y)$

$y = \tan^{-1}(x)$ is equivalent to $x = \tan(y)$

Inverse Properties

$$\cos\left(\cos^{-1}(x)\right) = x \qquad \cos^{-1}\left(\cos(\theta)\right) = \theta$$

$$\sin\left(\sin^{-1}(x)\right) = x \qquad \sin^{-1}\left(\sin(\theta)\right) = \theta$$

$$\tan\left(\tan^{-1}(x)\right) = x \qquad \tan^{-1}\left(\tan(\theta)\right) = \theta$$

Domain and Range

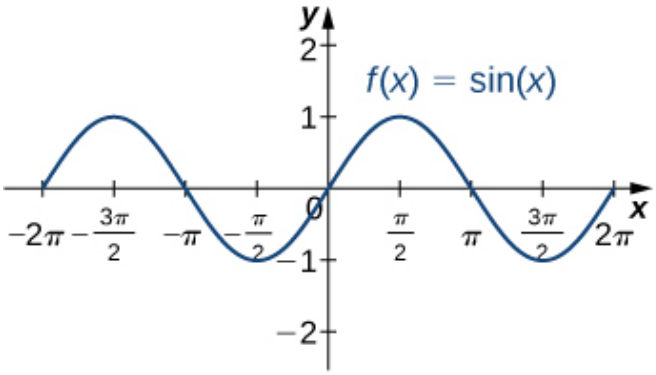
Function	Domain	Range
$y = \sin^{-1}(x)$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$
$y = \cos^{-1}(x)$	$-1 \le x \le 1$	$0 \le y \le \pi$
$y = \tan^{-1}(x)$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$

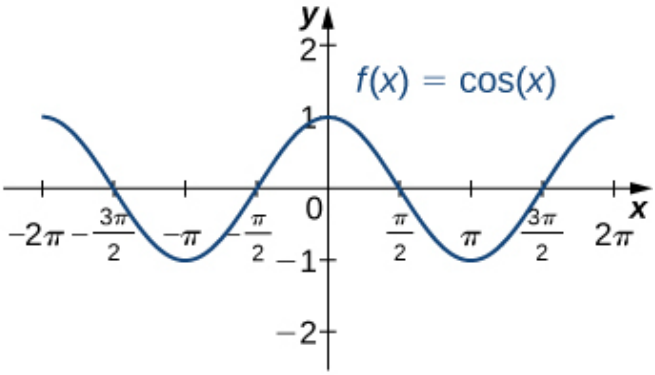
Alternate Notation

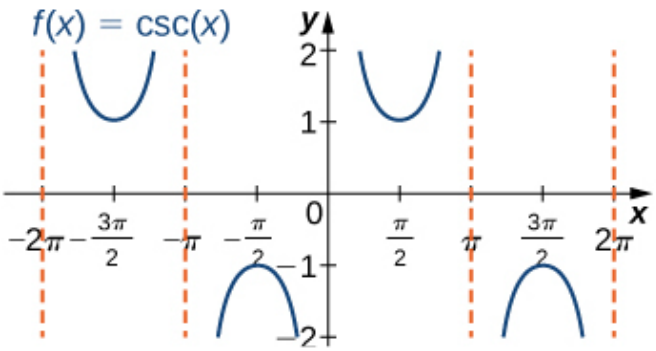
$\sin^{-1}(x) = \arcsin(x) \neq \csc(x)$

$\cos^{-1}(x) = \arccos(x) \neq \sec(x)$

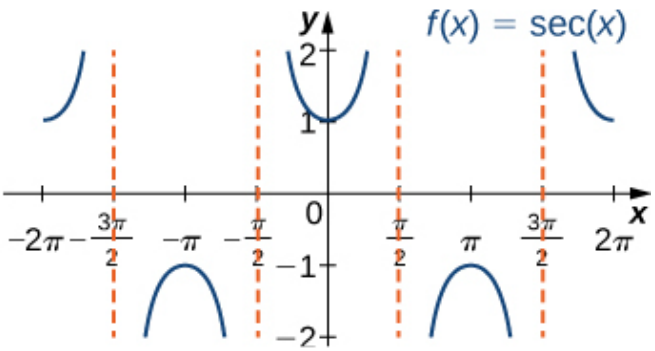
$\tan^{-1}(x) = \arctan(x) \neq \cot(x)$



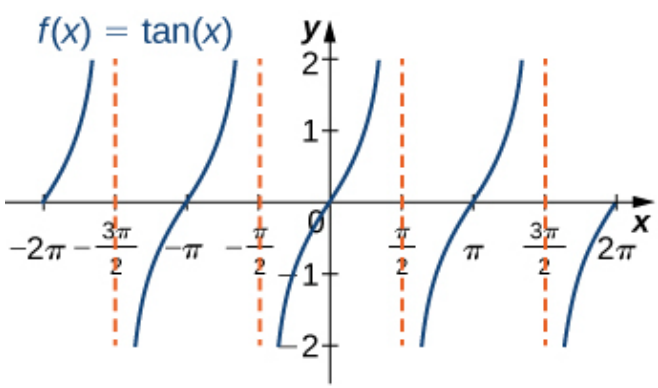




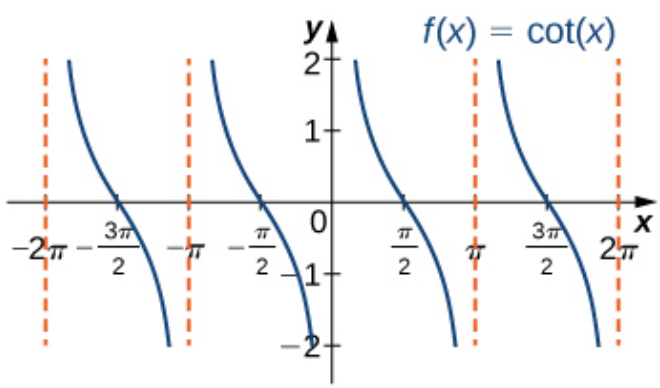
Asymptotes at:



Asymptotes at:



Asymptotes at:



Asymptotes at:

Pythagorean Identities

$\sin^2(\theta) + \cos^2(\theta) = 1$

$\tan^2(\theta) + 1 = \sec^2(\theta)$

$1 + \cot^2(\theta) = \csc^2(\theta)$

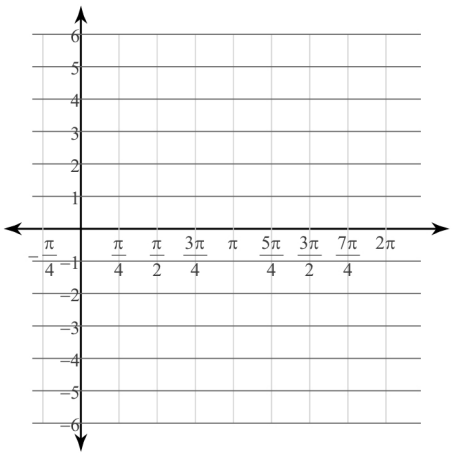
Even/Odd Formulas

$\sin(-\theta) = -\sin(\theta)$ $\quad \csc(-\theta) = -\csc(\theta)$

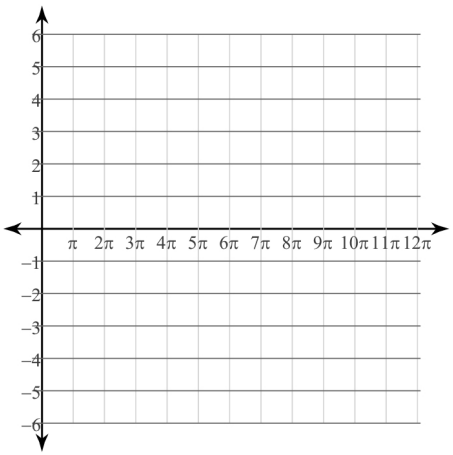
$\cos(-\theta) = \cos(\theta)$ $\quad \sec(-\theta) = \sec(\theta)$

$\tan(-\theta) = -\tan(\theta)$ $\quad \cot(-\theta) = -\cot(\theta)$

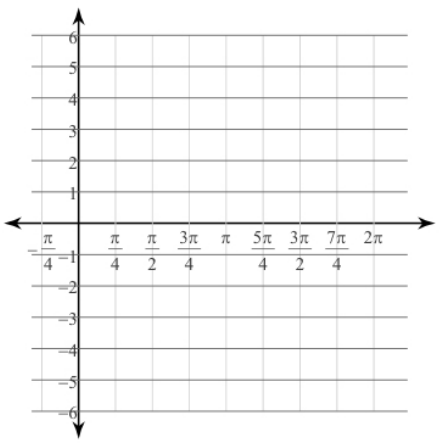
3) $y = 2\sin\left(-3\theta - \frac{\pi}{2}\right) + 2$



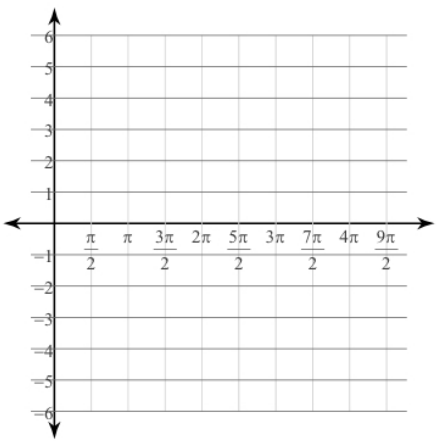
4) $y = \cos\left(\frac{\theta}{4} + \frac{\pi}{4}\right) - 2$



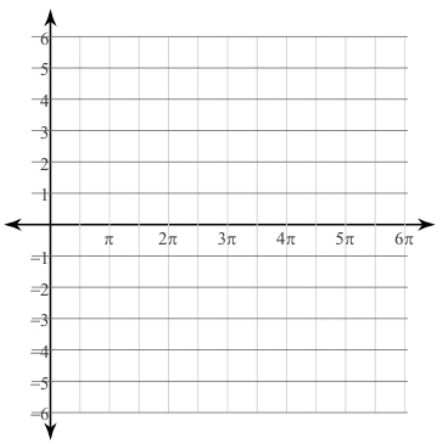
$y = \tan\left(2\theta - \frac{11\pi}{6}\right) - 1$



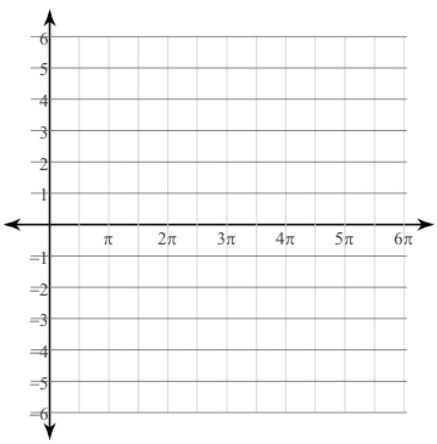
$y = 2\cot\left(\frac{\theta}{3} + \frac{5\pi}{6}\right)$



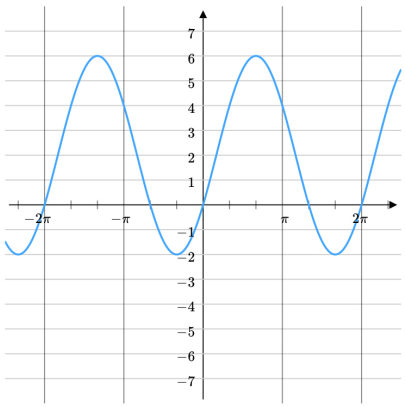
$y = \frac{1}{2} \cdot \csc\left(\frac{\theta}{2} + \frac{3\pi}{4}\right)$



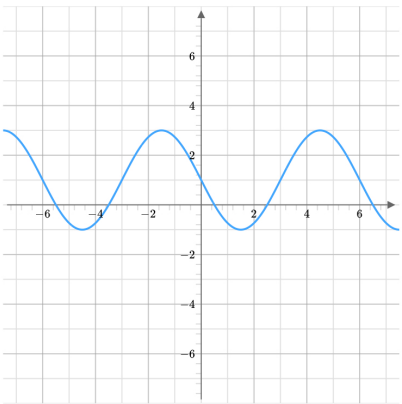
$y = 3\sec\left(\frac{\theta}{2} - \frac{3\pi}{4}\right)$



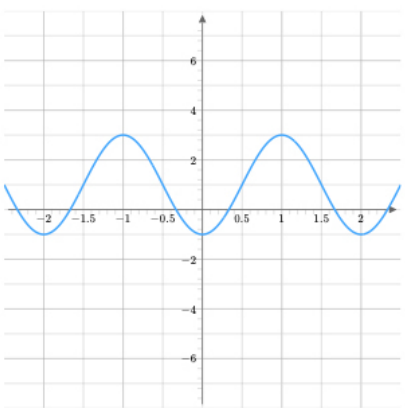
1. Write an equation of a sine function in the form $y = A\sin(Bx + C) + D$ for the following graph.



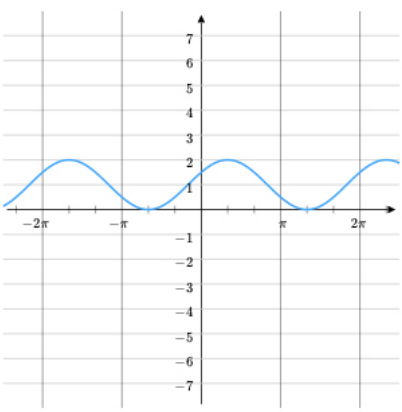
2. Write an equation of a sine function in the form $y = A\sin(Bx + C) + D$ for the following graph.



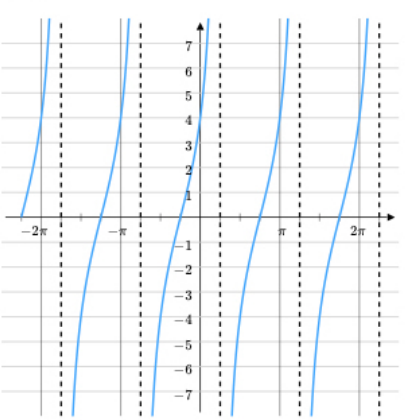
3. Write an equation of a cosine function in the form $y = A\cos(Bx + C) + D$ for the following graph.



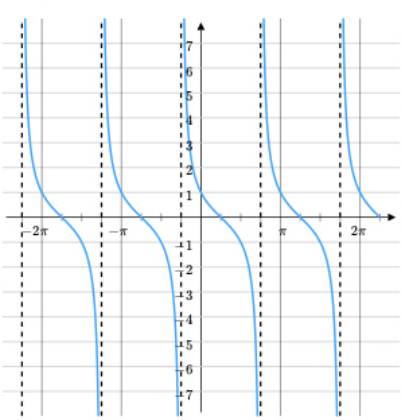
4. Write an equation of a cosine function in the form $y = A\cos(Bx + C) + D$ for the following graph.



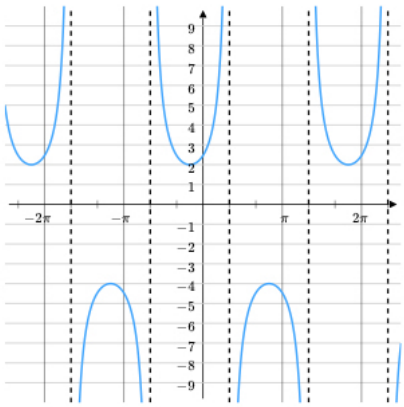
5. Write an equation of a tangent function in the form $y = A\tan(Bx + C) + D$ for the following graph.



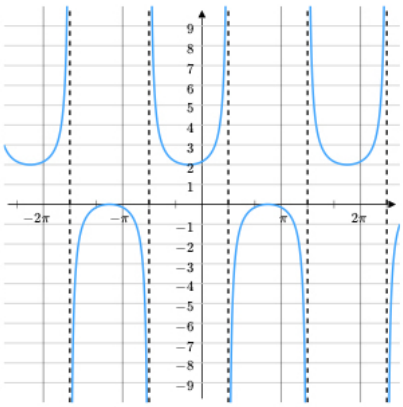
6. Write an equation of a tangent function in the form $y = A\tan(Bx + C) + D$ for the following graph.



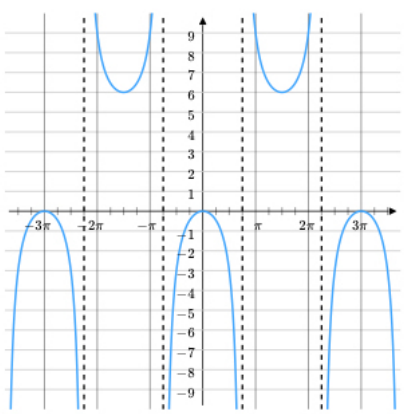
7. Write an equation of a cosecant function in the form $y = A\csc(Bx + C) + D$ for the following graph.



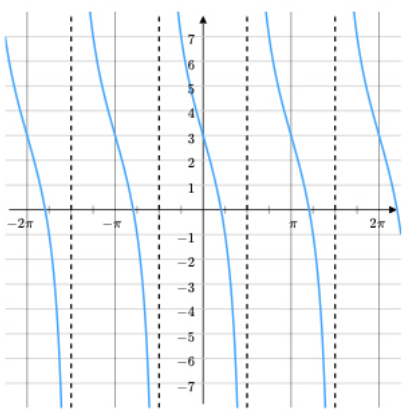
8. Write an equation of a cosecant function in the form $y = A\csc(Bx + C) + D$ for the following graph.



9. Write an equation of a secant function in the form $y = A\sec(Bx + C) + D$ for the following graph.



10. Write an equation of a cotangent function in the form $y = A\cot(Bx + C) + D$ for the following graph.



Convert each degree measure into radians and each radian measure into degrees.

$-\frac{\pi}{6}$

$-\frac{23\pi}{6}$

-30°

-930°

-210°

$\frac{\pi}{4}$

-160°

$-\frac{\pi}{3}$

$\frac{11\pi}{6}$

$\frac{17\pi}{12}$

In Exercises 1 – 20, find the exact value or state that it is undefined.

1. $\tan\left(\frac{\pi}{4}\right)$
2. $\sec\left(\frac{\pi}{6}\right)$
3. $\csc\left(\frac{5\pi}{6}\right)$
4. $\cot\left(\frac{4\pi}{3}\right)$
5. $\tan\left(-\frac{11\pi}{6}\right)$
6. $\sec\left(-\frac{3\pi}{2}\right)$
7. $\csc\left(-\frac{\pi}{3}\right)$
8. $\cot\left(\frac{13\pi}{2}\right)$
9. $\tan(117\pi)$
10. $\sec\left(-\frac{5\pi}{3}\right)$
11. $\csc(3\pi)$
12. $\cot(-5\pi)$
13. $\tan\left(\frac{31\pi}{2}\right)$
14. $\sec\left(\frac{\pi}{4}\right)$
15. $\csc\left(-\frac{7\pi}{4}\right)$
16. $\cot\left(\frac{7\pi}{6}\right)$
17. $\tan\left(\frac{2\pi}{3}\right)$
18. $\sec(-7\pi)$
19. $\csc\left(\frac{\pi}{2}\right)$
20. $\cot\left(\frac{3\pi}{4}\right)$

In Exercises 21 – 44, find all angles which satisfy the given equation.

21. $\sin(\theta)=\frac{1}{2}$
22. $\cos(\theta)=-\frac{\sqrt{3}}{2}$
23. $\sin(\theta)=0$
24. $\cos(\theta)=\frac{\sqrt{2}}{2}$
25. $\sin(\theta)=\frac{\sqrt{3}}{2}$
26. $\cos(\theta)=-1$
27. $\sin(\theta)=-1$
28. $\cos(\theta)=\frac{\sqrt{3}}{2}$
29. $\cos(\theta)=-1.001$
30. $\tan(\theta)=\sqrt{3}$
31. $\sec(\theta)=2$
32. $\csc(\theta)=-1$
33. $\cot(\theta)=\frac{\sqrt{3}}{3}$
34. $\tan(\theta)=0$
35. $\sec(\theta)=1$
36. $\csc(\theta)=2$
37. $\cot(\theta)=0$
38. $\tan(\theta)=-1$
39. $\sec(\theta)=0$
40. $\csc(\theta)=-\frac{1}{2}$
41. $\sec(\theta)=-1$
42. $\tan(\theta)=-\sqrt{3}$
43. $\csc(\theta)=-2$
44. $\cot(\theta)=-1$