Circuit Training – Rational and Polynomial Inequalities **Name:** KEY

Directions: Begin in cell #1. Read the question and do the work necessary to answer it. Circle your answer then search for it. When you find it, call this cell #2 and proceed in this manner until you complete the circuit by returning to the beginning.

Solve the inequalities and write the answers in interval notation.

Answer: $(-5,-1) \cup (1,2)$

#1

$$\frac{x-7}{x-1} < 0$$

Zeros: $x-7=0$

$x=7$

VA: $x-1=0$

$x=1$

$f(0) = \frac{-}{-} = +$

$f(2) = \frac{-}{+} = -$

$f(8) = \frac{+}{+} = +$

$(1,7)$

Answer: $[-6,-3) \cup (8,\infty)$

______ 4

$$x^2 - 4x > 32$$

$x^2-4x-32>0$

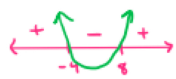
$(x-8)(x+4)>0$

$x=8,-4$

$f(-5) = (-)(-) = +$

$f(0) = (-)(+) = -$

$f(9) = (+)(+) = +$



$(-\infty,-4) \cup (8,\infty)$

Answer: $(-\infty,-4) \cup (8,\infty)$

______ 5

$$\frac{x+32}{x+6} \le 3\left(\frac{x+6}{x+6}\right)$$

Zeros: $x-7=0$

$x=7$

VA: $x+6=0$

$x=-6$

$f(-7) = \frac{+}{-} = -$

$f(0) = \frac{+}{+} = +$

$f(8) = \frac{-}{+} = -$

$\frac{x+32}{x+6} \le \frac{3x+18}{x+6}$

$\frac{x+32}{x+6} - \frac{3x+18}{x+6} \le 0$

$\frac{-2x+14}{x+6} \le 0$

$\frac{-2(x-7)}{x+6} \le 0$

$(-\infty,-6) \cup [7,\infty)$

Answer: $(-\infty,-6) \cup [7,\infty)$

______ 6

$$(x+5)(x-2)(x-1)(x+1) < 0$$

$x=-5,2,1,-1$

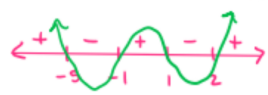
$f(-6) = (-)(-)(-)(-) = +$

$f(-2) = (+)(-)(-)(-) = -$

$f(0) = (+)(-)(-)(+) = +$

$f(1.5) = (+)(-)(+)(+) = -$

$f(3) = (+)(+)(+)(+) = +$



$(-5,-1) \cup (1,2)$

Answer: $(-\infty,\infty)$

______ 3

$$\frac{x+6}{x^2-5x-24} \ge 0$$

$\frac{x+6}{(x-8)(x+3)} \ge 0$

zeros: $x+6=0$

$x=-6$

VA: $(x-8)(x+3)=0$

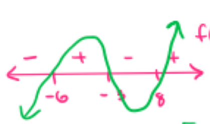
$x=8,-3$

$f(-7) = \frac{-}{(-)(-)} = -$

$f(-4) = \frac{+}{(-)(-)} = +$

$f(0) = \frac{+}{(-)(+)} = -$

$f(9) = \frac{+}{(+)(+)} = +$



$[-6,-3) \cup (8,\infty)$

Answer: $(1,7)$

______ 2

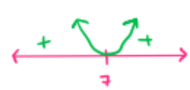
$$x^2 - 14x + 49 \ge 0$$

$(x-7)^2 \ge 0$

$x=7$

$f(6) = +$

$f(8) = +$



$(-\infty,\infty)$

3 Exercises

1. $2^x = 16$

$\log_2(2^x) = \log_2(16)$

$16 = 2^{4}$

$x = 4$

2. $3^x = 81$

$\log_3(3^x) = \log_3(81)$

$81 = 3^{4}$

$x = 4$

3. $4^x = 256$

$\log_4(4^x) = \log_4(256)$

$256 = 4^{4}$

$x = 4$

4. $5^x = 625$

$\log_5(5^x) = \log_5(625)$

$625 = 5^{4}$

$x = 4$

5. $10^x = 1000$

$\log(10^x) = \log(1000)$

$1000 = 10^{3}$

$x = 3$

6. $e^x = 20$

$\ln(e^x) = \ln(20)$

$x = \ln(20)$

7. $2^{2x-1} = 8$

$\log_2(2^{2x-1}) = \log_2(8)$

$8 = 2^{3}$

$2x-1 = 3$
$+1 \quad +1$
$\frac{2x}{2} = \frac{4}{2}$

$x = 4/2 = 2$

8. $9^{2x+1} = 729$

$\log_9(9^{2x+1}) \quad \log_9(729)$

$729 = 9^{3}$

$2x+1 = 3$
$-1 \quad -1$
$\frac{2x}{2} = \frac{2}{2}$

$x = \frac{2}{2} = 1$

9. $5^{3x-2} = 125$

$\log_5(5^{3x-2}) = \log_5(125)$

$125 = 5^{3}$

$3x-2 = 3$
$+2 \quad +2$
$\frac{3x}{3} = \frac{5}{3}$

$x = \frac{5}{3}$

10. $6^{x+2} = 216$

$\log_6(6^{x+2}) \quad \log_6(216)$

$216 = 6^{3}$

$x+2 = 3$
$-2 \quad -2$

$x = 1$

11. $7^{3x} = 343$

$\log_7(7^{3x}) = \log_7(343)$ $\quad 343 = 7^{3}$

$\frac{3x}{3} = \frac{3}{3}$

$x = 1$

12. $8^{x-1} = 4$

$\log_8(8^{x-1}) = \log_8(4)$ $\quad 4 = 8^{x} \rightarrow 2^2 = (2^3)^x = 2^{3x}$

$\log_2(2^2) = \log_2(2^{3x})$

$2 = 3x$

$\frac{2}{3} = x$

$x-1 = \frac{2}{3}$
$+1 \quad +1$ $\quad 1 = \frac{3}{3}$

$x = \frac{5}{3}$

13. $11^x = 121$

$\log_{11}(11^x) = \log_{11}(121)$

$x = 2$ $\quad 121 = 11^2$

14. $12^x = 144$

$\log_{12}(12^x)$ $\quad \log_{12}(144)$

$x = 2$ $\quad 144 = 12^2$

15. $15^x = 225$

$\log_{15}(15^x)$ $\quad \log_{15}(225)$

$x = 2$ $\quad 225 = 15^2$

4 Logarithmic Equations

Solve the following logarithmic equations:

1. $\log_2(x+1) = 3$

$2^{\log_2(x+1)} = 2^3$

$x + 1 = 8$ $\quad x = 7$

$-1 \quad -1$

2. $\log_3(x+2) = 2$

$3^{\log_3(x+2)} = 3^2$

$x + 2 = 9$ $\quad x = 7$

$-2 \quad -2$

3. $\log_4(x+3) = 1$

$4^{\log_4(x+3)} = 4^1$ $\quad x + 3 = \overset{5}{4}$ $\quad x = 1$

$-3 \quad -3$

4. $\log_5(x+4) = 2$

$5^{\log_5(x+4)} = 5^2$

$x+4 = 25$
$\quad -4 \quad -4$

$x = 21$

5. $\log_{10}(x+5) = 3$

$10^{\log_{10}(x+5)} = 10^3$

$x+5 = 1000$
$\quad -5 \quad -5$

$x = 995$

6. $\log_2(x-1) = 4$

$2^{\log_2(x-1)} = 2^4$

$x-1 = 16$
$\quad +1 \quad +1$

$x = 17$

7. $\log_3(x-2) = 3$

$3^{\log_3(x-2)} = 3^3$

$x-2 = 27$
$\quad +2 \quad +2$

$x = 29$

8. $\log_4(x-3) = 2$

$4^{\log_4(x-3)} = 4^2$

$x-3 = 16$
$\quad +3 \quad +3$

$x = 19$

9. $\log_5(x-4) = 1$

$5^{\log_5(x-4)} = 5^1$

$x-4 = 5$
$\quad +4 \quad +4$

$x = 9$

10. $\log_{10}(x-5)=0$

$$10^{\log_{10}(x-5)} = 10^0$$

$$x-5=1 \qquad x=6$$
$$+5 \quad +5$$

11. $\ln(x+1)=2$

$$e^{\ln(x+1)}=e^2$$

$$x+1=e^2 \qquad x=e^2-1$$
$$-1 \quad -1$$

12. $\ln(x-1)=1$

$$e^{\ln(x-1)}=e^1$$

$$x-1=e \qquad x=e+1$$
$$+1 \quad +1$$

13. $\log_2(x+1)+\log_2(x-1)=4$

$$2^{\log_2(x+1)+\log_2(x-1)}=2^4$$

$$\left(2^{\log_2(x+1)}\right)\left(2^{\log_2(x-1)}\right)=16$$

$$(x+1)(x-1)=16$$

	x	$+1$
x	x^2	$\cancel{+x}$
-1	$\cancel{-x}$	-1

$$x^2-1=16$$
$$+1 \quad +1$$

$$x=\sqrt{17}$$

14. $\log_3(x+2)-\log_3(x-2)=2$

$$\log_3\left(\frac{x+2}{x-2}\right)=2$$

$$3^{\log_3\left(\frac{x+2}{x-2}\right)}=3^2$$

$$\frac{x+2}{x-2}=9$$

$$x+2=9x-18$$
$$+18 \quad +18$$

$$x+20=9x$$
$$-x \quad -x$$

$$8x=20$$

$$x=\frac{20}{8}=\frac{5}{2}$$

15. $\ln(x+3)-\ln(x-3)=1$

$$\ln\left(\frac{x+3}{x-3}\right)=1$$

$$e^{\ln\left(\frac{x+3}{x-3}\right)}=e^1$$

$$\frac{x+3}{x-3}=e$$

$$x+3=ex-3e$$
$$+3e \quad +3e$$

$$x+3+3e=ex$$
$$-x \quad -x$$

$$3+3e=ex-x$$

$$3+3e=x(e-1)$$

$$\frac{3+3e}{e-1}=x$$