
WELCOME TO KEIRA ROBERTS' MATH 2412 SI

Please sign in on the orange
sheet at the front, grab a copy
of the worksheet, and get
started on today's warmup.



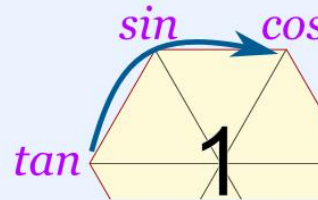
MAGIC HEXAGON ACTIVITY

Building It: The Quotient Identities

Start with:

$$\tan(x) = \sin(x) / \cos(x)$$

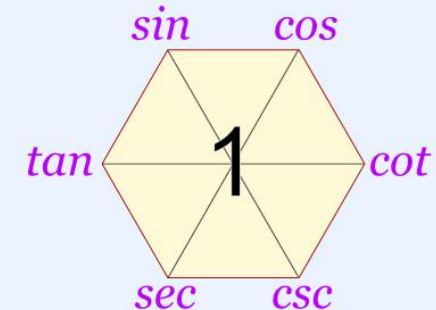
$$\tan(x) = \frac{\sin(x)}{\cos(x)}$$



To help you remember think "tsc !"

Then add:

- cot (which is **co**tangent) on the opposite side of the hexagon to tan
- csc (which is **co**secant) next, and
- sec (which is secant) last



To help you remember: the "co" functions are all on the right

OK, we have now built our hexagon, what do we get out of it?

Well, we can now follow "around the clock" (either direction) to get all the "Quotient Identities":

Clockwise

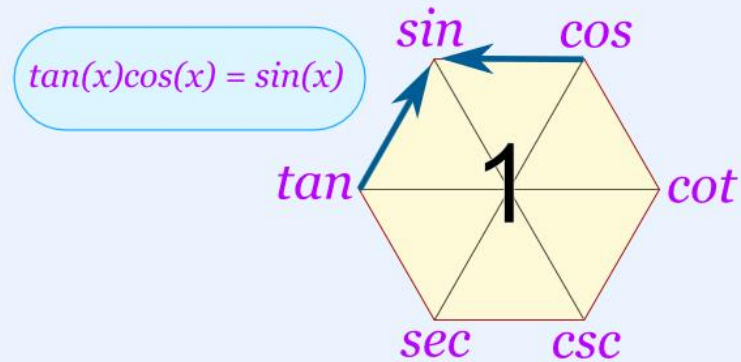
- $\tan(x) = \sin(x) / \cos(x)$
- $\sin(x) = \cos(x) / \cot(x)$
- $\cos(x) = \cot(x) / \csc(x)$
- $\cot(x) = \csc(x) / \sec(x)$
- $\csc(x) = \sec(x) / \tan(x)$
- $\sec(x) = \tan(x) / \sin(x)$

Counterclockwise

- $\cos(x) = \sin(x) / \tan(x)$
- $\sin(x) = \tan(x) / \sec(x)$
- $\tan(x) = \sec(x) / \csc(x)$
- $\sec(x) = \csc(x) / \cot(x)$
- $\csc(x) = \cot(x) / \cos(x)$
- $\cot(x) = \cos(x) / \sin(x)$

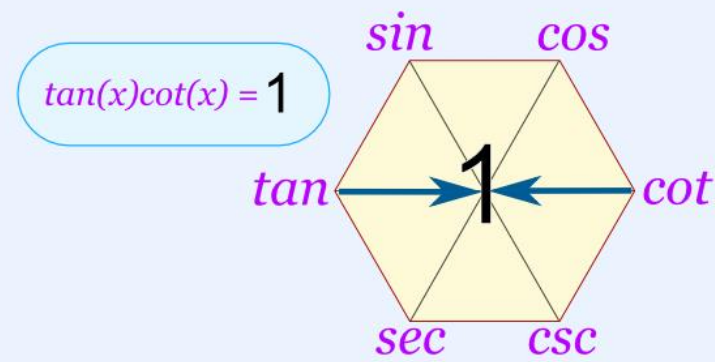
Product Identities

The hexagon also shows that a function **between** any two functions is equal to them multiplied together (if they are opposite each other, then the "1" is between them):



Example:

$$\tan(x)\cos(x) = \sin(x)$$



Example:

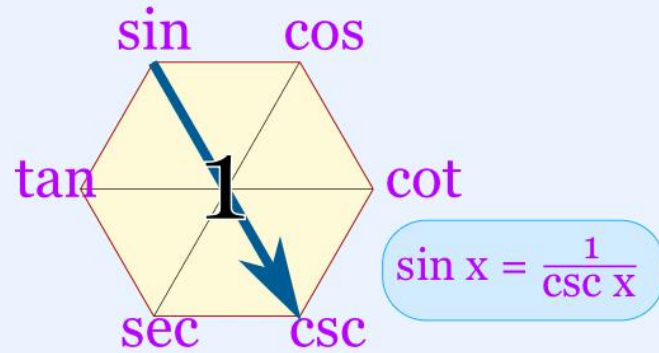
$$\tan(x)\cot(x) = 1$$

Some more examples:

- $\sin(x)\csc(x) = 1$
- $\tan(x)\csc(x) = \sec(x)$
- $\sin(x)\sec(x) = \tan(x)$

But Wait, There is More!

You can also get the "Reciprocal Identities", by going "through the 1"



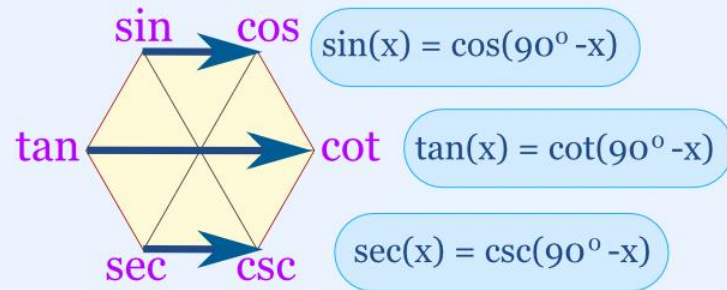
Here you can see that **$\sin(x) = 1 / \csc(x)$**

Here is the full set:

- $\sin(x) = 1 / \csc(x)$
- $\cos(x) = 1 / \sec(x)$
- $\cot(x) = 1 / \tan(x)$
- $\csc(x) = 1 / \sin(x)$
- $\sec(x) = 1 / \cos(x)$
- $\tan(x) = 1 / \cot(x)$

Bonus!

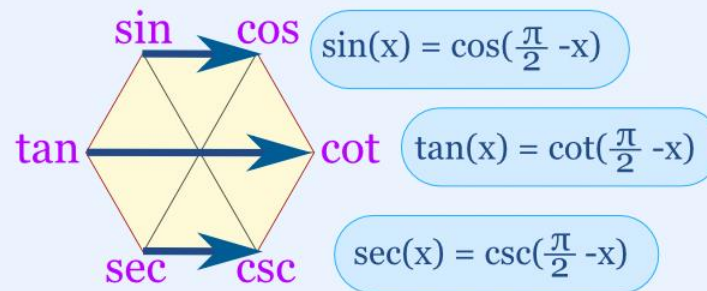
AND we also get these co-function identities:



Examples:

- $\sin(30^\circ) = \cos(60^\circ)$
- $\tan(80^\circ) = \cot(10^\circ)$
- $\sec(40^\circ) = \csc(50^\circ)$

Or, if you prefer, in radians :



Examples:

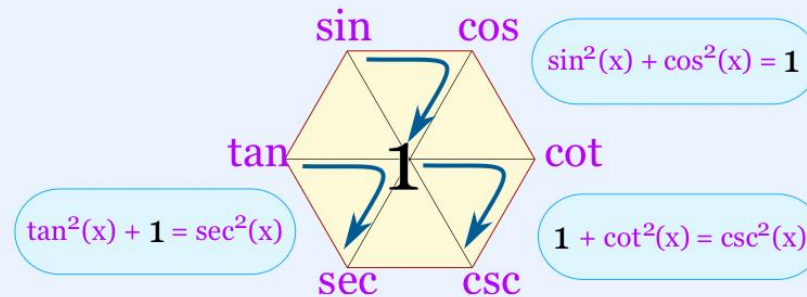
- $\sin(0.1\pi) = \cos(0.4\pi)$
- $\tan(\pi/4) = \cot(\pi/4)$
- $\sec(\pi/3) = \csc(\pi/6)$

Double Bonus: The Pythagorean Identities

The Unit Circle shows us that

$$\sin^2 x + \cos^2 x = 1$$

The magic hexagon can help us remember that, too, by going clockwise around any of these three triangles:



And we have:

- $\sin^2(x) + \cos^2(x) = 1$
- $1 + \cot^2(x) = \csc^2(x)$
- $\tan^2(x) + 1 = \sec^2(x)$

You can also travel counterclockwise around a triangle, for example:

- $1 - \cos^2(x) = \sin^2(x)$

SIMPLIFYING TRIG WORKSHEET
