Equation of a Circle - Skills

1) Find the centre and radius of each of the following circle equations

a) $x^2 + y^2 = 36$

b) $(x+2)^2 + (y+5)^2 = 64$

c) $(x-4)^2 + (y+7)^2 = 25$

d) $(x-2)^2 + (y-5)^2 = 49$

e) $(x+3)^2 + y^2 = 16$

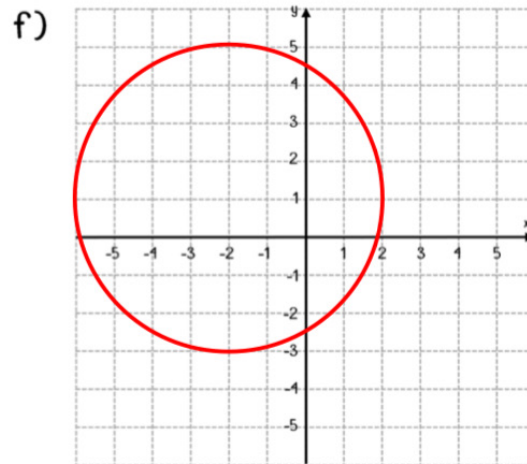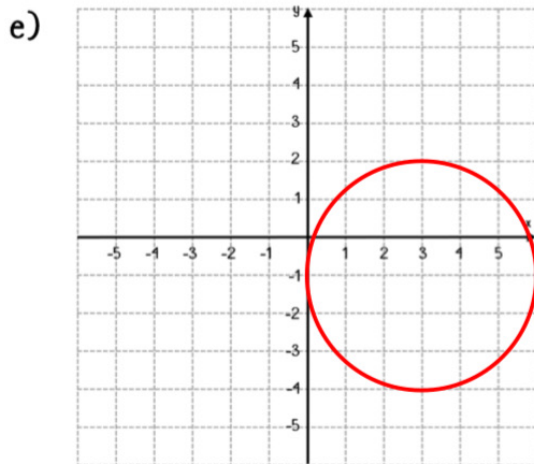f) $x^2 + y^2 + 10x + 8y - 40 = 0$

2) Find the equation of each circle below in the form $(x - h)^2 + (y - k)^2 = r^2$

a) Centre = (8, 7)   Radius = 3
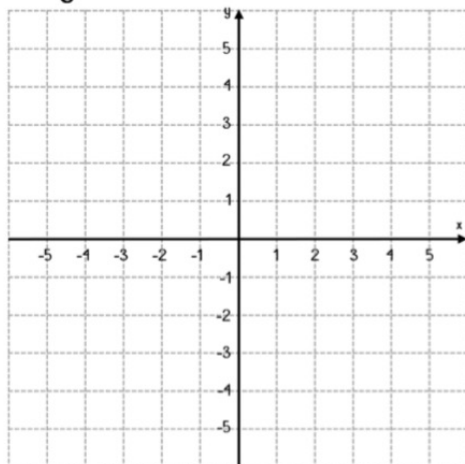
b) Centre = (-6, 2)   Radius = 10

c) Centre = (-3, -5)   Radius = 12

d) Centre = (-0.5, 3)   Radius = $\sqrt{3}$
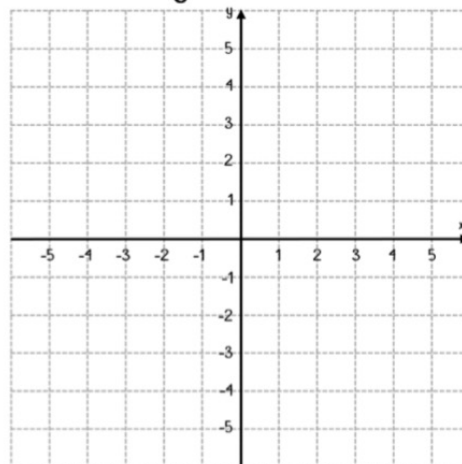
e)

f)

3) Draw the following equations on the on the cartesian grids below

a) $x^2 + y^2 = 25$

b) $(x - 1)^2 + (y + 2)^2 = 4$

Find the asymptotes and the x & y intercepts to match each equation to its graph.

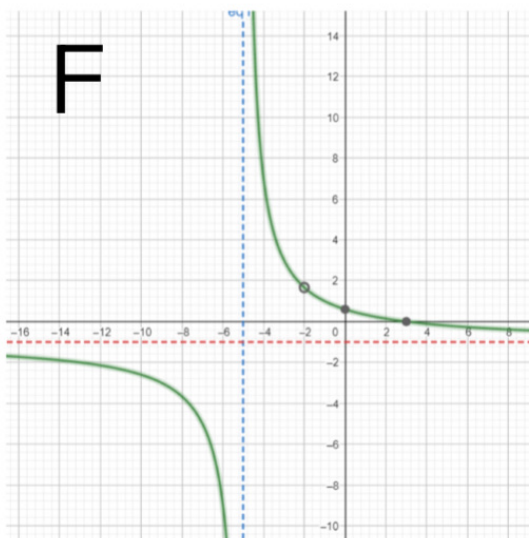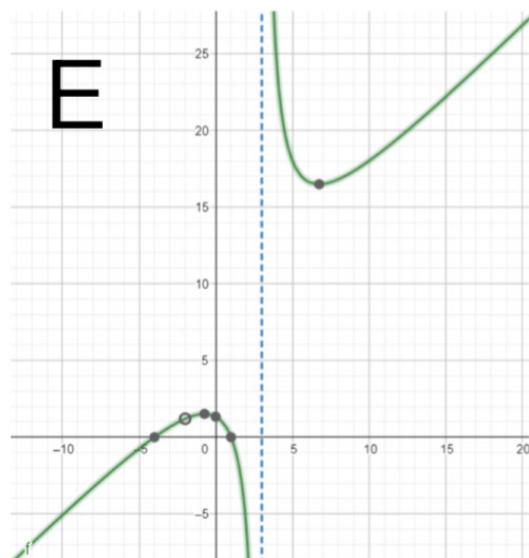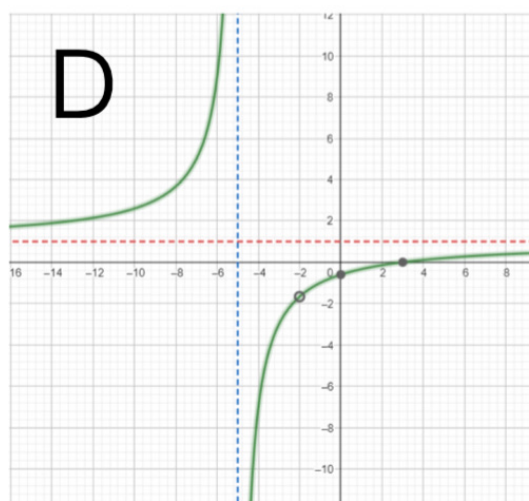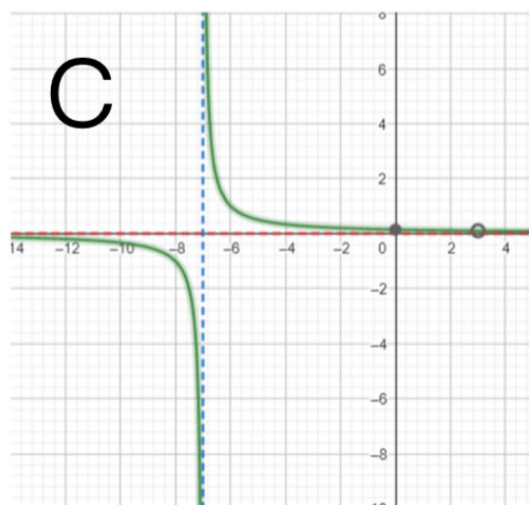$$\frac{x^2-x-6}{x^2+7x+10} \qquad \frac{x^2-x-6}{-x^2-7x-10} \qquad \frac{x^2+6x-7}{2x^2+8x-42}$$

$$\frac{x-3}{x^2+4x-21} \qquad \frac{(x+2)(x-1)(x+4)}{(x-3)(x+2)} \qquad \frac{2x^2+12-14}{x^2+4x-21}$$

A

B

C

D

E

F
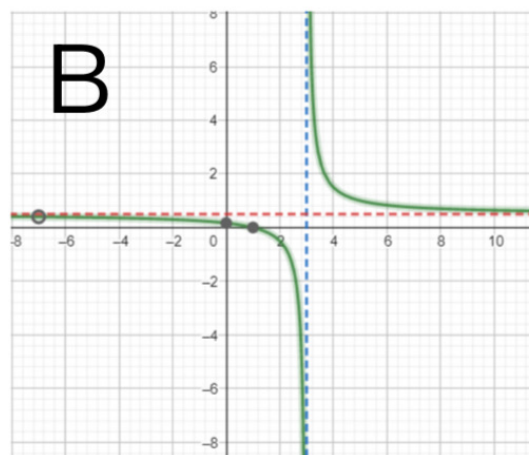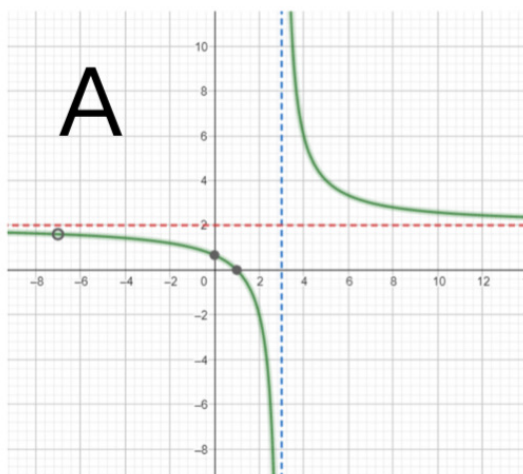
Piecewise Functions

Graph all of the following functions in their corresponding colors to create a picture below.

Blue:

$$y = \begin{cases} -\sqrt{-x-\pi} - 2 & -10 \le x \le -\pi \\ \sin x - 2 & -\pi < x \le 10 \end{cases}$$

Gold:

$$y = \begin{cases} \sqrt{x+\pi} - 2 & -10 \le x \le -\pi \\ 10 - \sqrt{25 - (x-10)^2} & 5 \le x \le 10 \end{cases}$$

Black:

$$y = \begin{cases} 3 + \sqrt{-x-1} & -2 \le x \le -1 \\ 3 + \sqrt{x+1} & -1 < x \le 0 \end{cases}$$

$$\begin{cases} 6 + \sqrt{-x+2} & 1 \le x \le 2 \\ 6 + \sqrt{x-2} & 2 < x \le 3 \end{cases}$$

$$\begin{cases} 4 + \sqrt{-x-1} & 3 \le x \le 4 \\ 4 + \sqrt{x-4} & 4 < x \le 5 \end{cases}$$
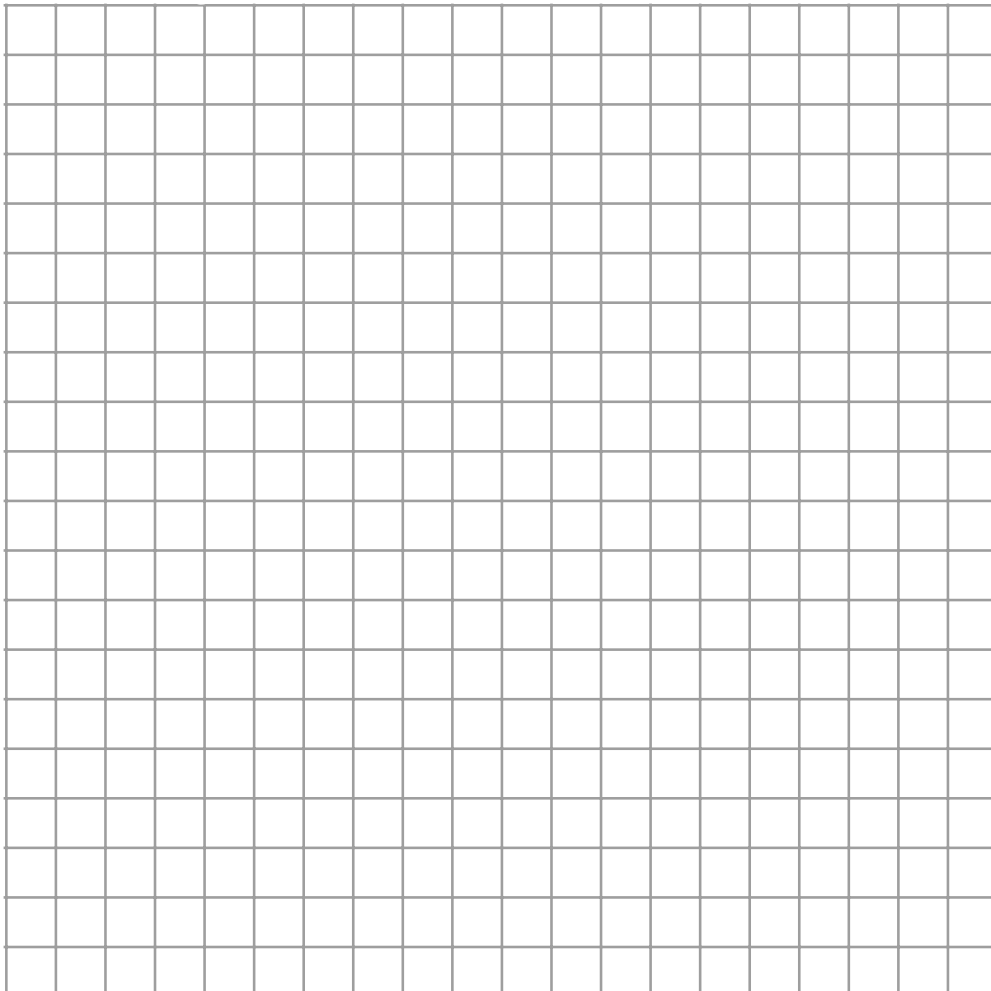
$$2^{2x-1} = 8$$

$$9^{2x+1} = 729$$

$$5^{3x-2} = 125$$

$$6^{x+2} = 216$$

$$7^{3x} = 343$$

$$8^{x-1} = 4$$

$$\log_5(x + 4) = 2$$

$$\log_{10}(x + 5) = 3$$

$$\log_2(x - 1) = 4$$

$$\log_3(x - 2) = 3$$

$$\log_4(x - 3) = 2$$

$$\log_5(x - 4) = 1$$

Answer: $(-5, -4) \cup (1, \infty)$

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$$\frac{x}{2(x+2)} - \frac{1}{x+2} \leq \frac{12}{x(x+2)}$$

Answer: $[-4, -2) \cup (0, 6]$

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$$\frac{x+1}{2x^2 + 5x - 3} \geq \frac{x}{4x^2 - 1}$$

Answer: $(4, 19]$

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$$\frac{x^2 - 11x - 12}{x+4} < 0$$

Answer: $\{-8\} \cup \{-7\} \cup [-5, \infty)$

____

$$-\frac{7}{x+5} \leq -\frac{8}{x+6}$$

Answer: $[-5, 5) \cup (6, \infty)$

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$$(x+8)^2 (x+5)(x+7)^2 \geq 0$$

Answer: $\left(-3, -\frac{1}{2}\right) \cup \left(\frac{1}{2}, \infty\right)$

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$$-\frac{10}{x-5} \geq -\frac{11}{x-6}$$

Answer: $(-\infty, -6) \cup (-5, 2]$

____

$$\frac{2x+7}{x-4} \geq 3$$

Answer: $(-\infty, -4) \cup (-1, 12)$

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$$\frac{1}{x+5} + \frac{5}{x-1} > 0$$

- You learned that quantities that grow or decay exponentially increase or decrease at a constant percent rate.

- Some quantities have a constant doubling time or half-life.

- When the doubling time, d, or half-life, h, is known, the relationship between the initial amount, A_0, and the amount A after time t can be modelled by these equations:

  Exponential Growth

  - Doubling time is the time it takes for a population to double in size.

  $$A = A_0 (2)^{\frac{t}{d}}$$

  Where: A = final quantity
  A_0 = initial amount
  2 = indicates doubling
  t = time
  d = doubling time

  Exponential Decay

  - Half-Life is the time it takes for a quantity to decay to half the original amount.

  $$A = A_0 (0.5)^{\frac{t}{h}}$$

  Where: A = final quantity
  A_0 = initial amount
  0.5 = indicates half-life
  t = time
  h = half-life

PRACTICE:

1. Caffeine has a half-life of approximately 5 h. Suppose you drink a cup of coffee that contains 200 mg of caffeine. How long will it take until there is less than 10 mg of caffeine left in your bloodstream? Give your answer to 1 decimal place.

2. Tritium, a radioactive gas that builds up in CANDU nuclear reactors, is collected, stored in pressurized gas cylinders, and sold to research laboratories. Tritium decays into helium over time. Its half-life is about 12.3 years.

   a) Write an equation that gives the mass of tritium remaining in a cylinder that originally contained 500 g of tritium.

   b) Estimate the time it takes until less than 5 g of tritium is present.

3. A colony of bacteria doubles in size every 20 min. How long will it take for a colony of 20 bacteria to grow to a population of 10 000?

4. An archaeologist uses radiocarbon dating to determine the age of a Viking ship. Suppose that a sample that originally contained 100 mg of Carbon-14 now contains 85 mg of Carbon-14. What is the age of the ship to the nearest hundred years? The half life of Carbon-14 is 5,730 years.

5. A bacteria culture starts with 6 500 bacteria. After 2.5 hours, there are 208 000 bacteria present. What is the length of the doubling period?

6. A bacteria culture doubles every 0.25 hours. At time 1.25 hours, there are 40 000 bacteria present. How many bacteria were present initially?

7. A sodium isotope, Na^{24}, has a half-life of 15 hours. Determine the amount of sodium that remains from a 4 g sample after:

   a) 45 hours   b) 100 hours   c) 5 days

8. The 50 cent Bluenose is one of Canada's most famous postage stamps. In 1930 it could be bought at the post office for $0.50. In 2000, a stamp in excellent condition was sold at an auction for $512. Determine the doubling time for the stamp's value.

9. Strontium-90 has a half-life of 25 years. How long would it take for 40 mg of it to decay to:

   a) 20 mg   b) 1.25 mg