Apply logarithmic functions
Answer key

1. The decibel level, D, of a sound with intensity I, as measured in watts per square inch, is given by $D = 10 \cdot \log\left(\frac{I}{10^{-12}}\right)$.

   What is the decibel level of a conversation with intensity 10^{-6} watts per square inch?

   60

2. The decibel level, D, of a sound with intensity I, as measured in watts per square inch, is given by $D = 10 \cdot \log\left(\frac{I}{10^{-12}}\right)$.

   What is the decibel level of a conversation with intensity 10^{-8} watts per square inch?

   40

3. The decibel level, D, of a sound with intensity I, as measured in watts per square inch, is given by $D = 10 \cdot \log\left(\frac{I}{10^{-12}}\right)$.

   What is the decibel level of a conversation with intensity 10^{-10} watts per square inch?

   20

4. The decibel level, D, of a sound with intensity I, as measured in watts per square inch, is given by $D = 10 \cdot \log\left(\frac{I}{10^{-12}}\right)$.

   What is the decibel level of a conversation with intensity 10^{-7} watts per square inch?

   50

5. A virus takes 5 days to grow from 50 to 120 specimens. How many days will it take to grow from 50 to 150 specimens? Round to the nearest whole number.

   6

6. A virus takes 2 days to grow from 50 to 170 specimens. How many days will it take to grow from 50 to 620 specimens? Round to the nearest whole number.

   4

7. A virus takes 13 days to grow from 130 to 190 specimens. How many days will it take to grow from 130 to 430 specimens? Round to the nearest whole number.

   41

8. A rocket engine is test fired, bringing its temperature to $4420^\circ F$. The air temperature is $69^\circ F$. Then 19 minutes after it is shut off, its temperature is $3846^\circ F$.

   $1.$ Write the exponential function describing the temperature in $^\circ F$ as a function of time in minutes.
   $2.$ How many minutes will it take to cool from the initial $4420^\circ F$ to $150^\circ F$?

   Round values to three decimal places.

   $f(t) = 4351 \cdot 0.993^t + 69$
   $time = 564.204$

9. A coffee pot brings the temperature of the coffee to $178^\circ F$. The room temperature is $69^\circ F$. After 8 minutes, the coffee temperature is $164^\circ F$.

   $1.$ Write the exponential function describing the temperature in $^\circ F$ as a function of time in minutes.
   $2.$ How many minutes will it take to cool from the initial $178^\circ F$ to $150^\circ F$?

   Round values to three decimal places.

   $f(t) = 109 \cdot 0.983^t + 69$
   $time = 17.325$

10. A rocket engine is test fired, bringing its temperature to $5610^\circ F$. The air temperature is $54^\circ F$. Then 15 minutes after it is shut off, its temperature is $5046^\circ F$.

    $1.$ Write the exponential function describing the temperature in $^\circ F$ as a function of time in minutes.
    $2.$ How many minutes will it take to cool from the initial $5610^\circ F$ to $144^\circ F$?

    Round values to three decimal places.

    $f(t) = 5556 \cdot 0.993^t + 54$
    $time = 588.668$

Name: Key

$(x-h)^2+(y-k)^2=r^2$

Equation of a Circle - Skills

1) Find the centre and radius of each of the following circle equations

a) $x^2 + y^2 = 36$

$(x-0)^2+(y-0)^2=6^2$

center $(0,0)$ $r=6$

b) $(x+2)^2 + (y+5)^2 = 64$

$(x--2)^2+(y--5)^2=8^2$

Center $(-2,-5)$ $r=8$

c) $(x-4)^2 + (y+7)^2 = 25$

$(x-4)^2+(y--7)^2=5^2$

Center $(4,-7)$ $r=5$

d) $(x-2)^2 + (y-5)^2 = 49$

$(x-2)^2+(y-5)^2=7^2$

center $(2,5)$ $r=7$

e) $(x+3)^2 + y^2 = 16$

$(x--3)^2+(y-0)^2=4^2$

center $(-3,0)$ $r=4$

f) $x^2 + y^2 + 10x + 8y - 40 = 0$

$x^2+10x+y^2+8y=40$

$(x+5)^2+(y+4)^2=49$

$(x--5)^2+(y--4)^2=7^2$

center $(-5,-4)$ $r=7$

2) Find the equation of each circle below in the form $(x-h)^2 + (y-k)^2 = r^2$

a) Centre = (8, 7) Radius = 3

$(x-8)^2+(y-7)^2=9$

b) Centre = (-6, 2) Radius = 10

$(x+6)^2+(y-2)^2=100$

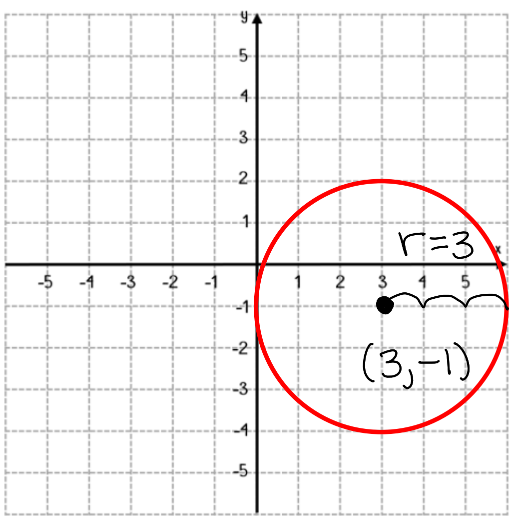
c) Centre = (-3, -5) Radius = 12

$(x+3)^2+(y+5)^2=144$

d) Centre = (-0.5, 3) Radius = $\sqrt{3}$

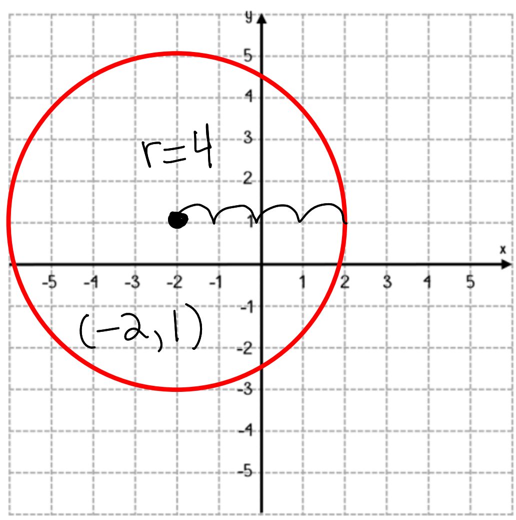
$(x+0.5)^2+(y-3)^2=3$

e)



$(x-3)^2+(y+1)^2=9$

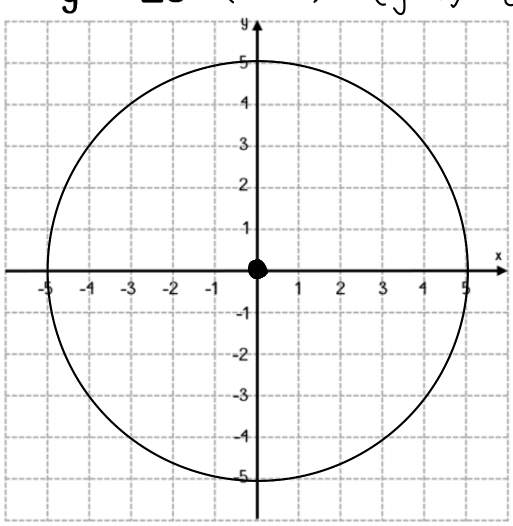
f)



$(x+2)^2+(y-1)^2=16$

3) Draw the following equations on the on the cartesian grids below

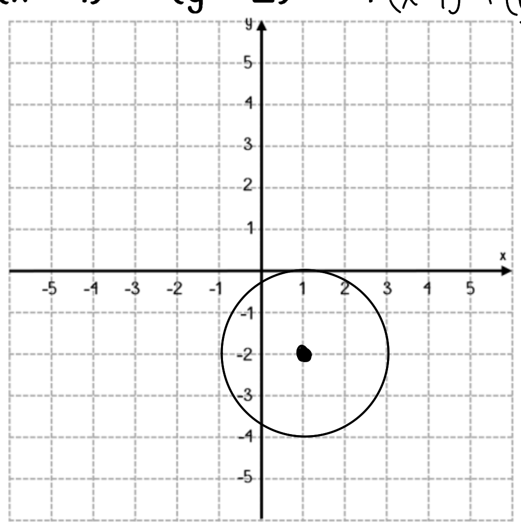
a) $x^2 + y^2 = 25$ $(x-0)^2+(y-0)^2=5^2$



C=(0,0)

r=5

b) $(x-1)^2 + (y+2)^2 = 4$ $(x-1)^2+(y--2)^2=2^2$



C=(1,-2)

r=2

$\frac{x^2-x-6}{x^2+7x+10}$ D

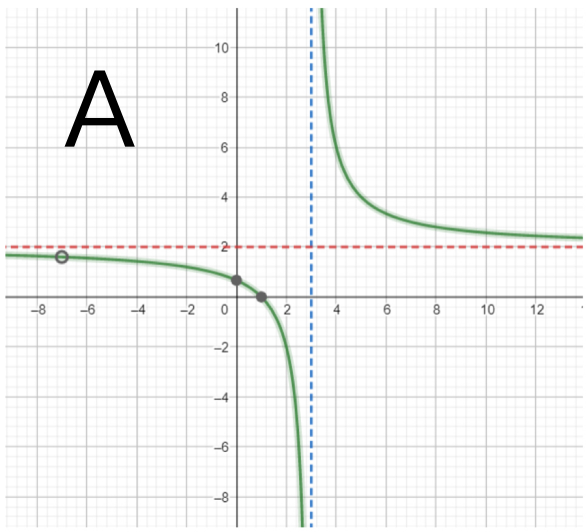
$\frac{x^2-x-6}{-x^2-7x-10}$ F

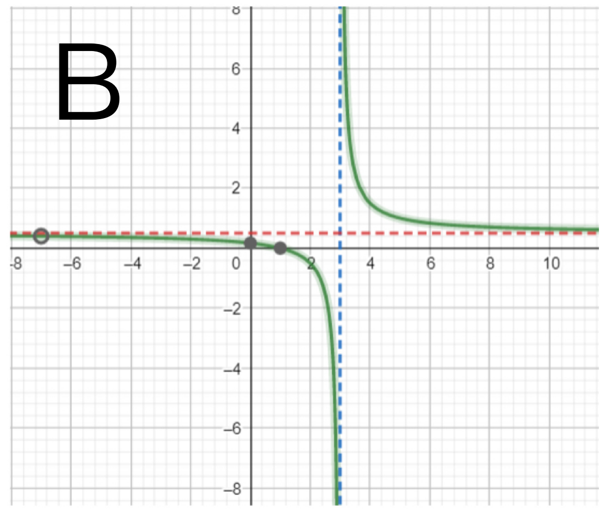
$\frac{x^2+6x-7}{2x^2+8x-42}$ B

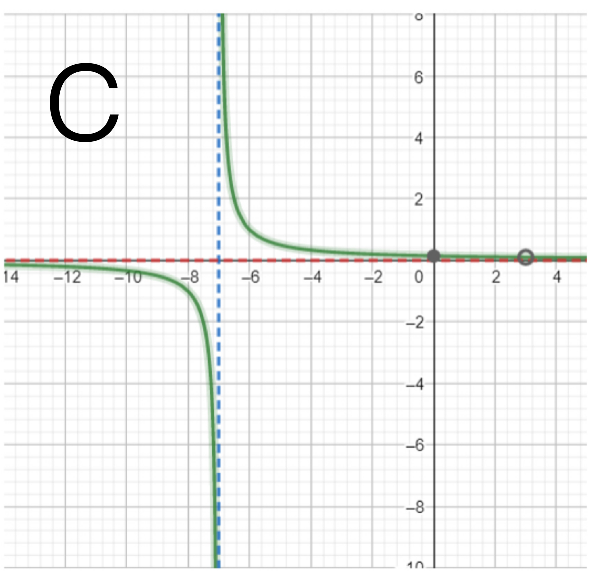
$\frac{x-3}{x^2+4x-21}$ C

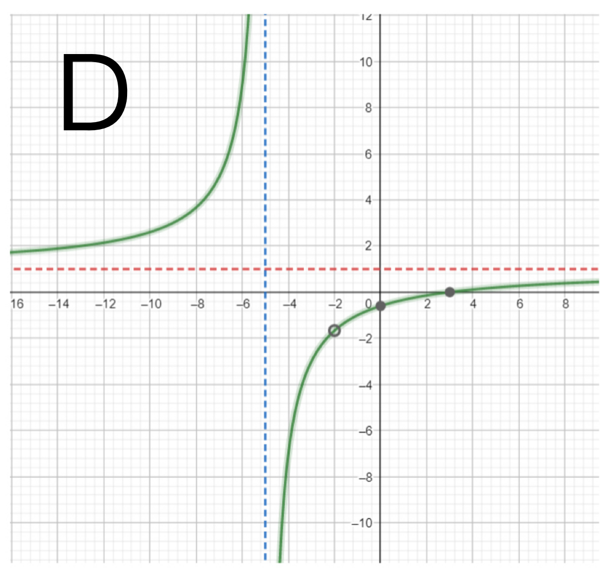
$\frac{(x+2)(x-1)(x+4)}{(x-3)(x+2)}$ E

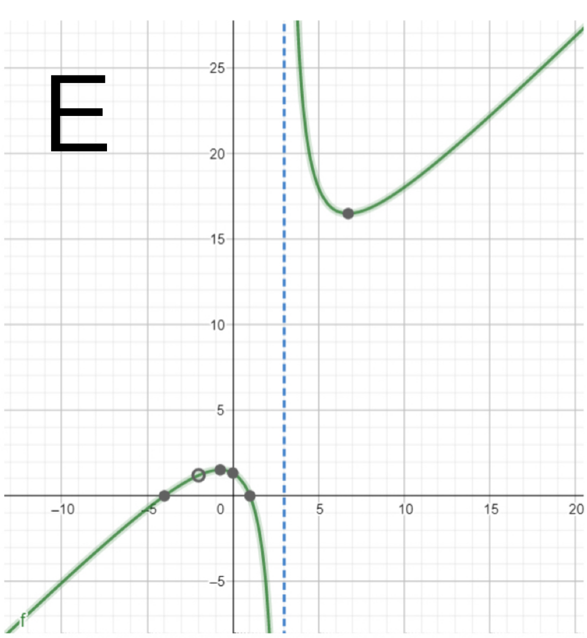
$\frac{2x^2+12x-14}{x^2+4x-21}$ A

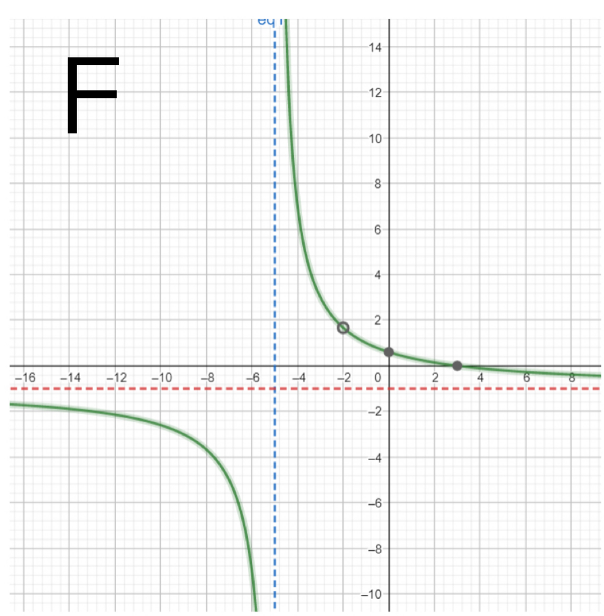


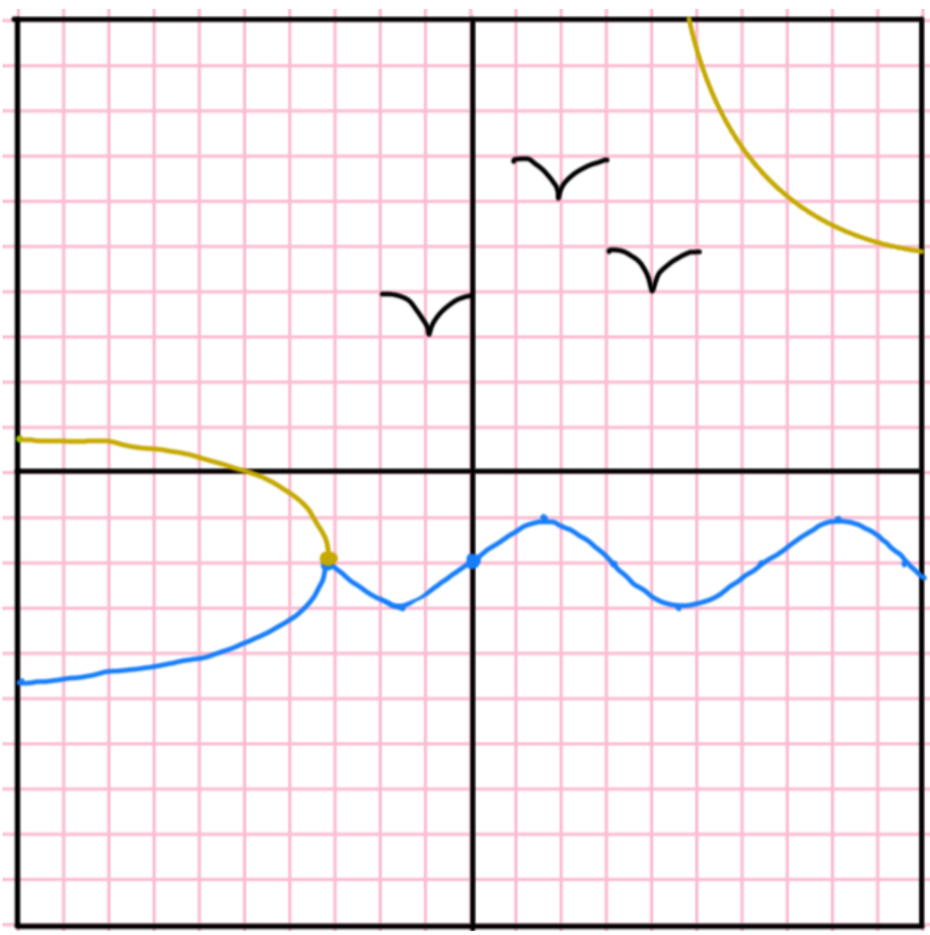












7. $2^{2x-1} = 8$

$\log_2(2^{2x-1}) = \log_2(8)$

$8 = 2^{3}$

$2x-1 = 3$

$+1 \quad +1$

$\frac{2x}{2} = \frac{4}{2}$

$x = 4/2 = 2$

8. $9^{2x+1} = 729$

$\log_9(9^{2x+1}) \quad \log_9(729)$

$729 = 9^{3}$

$2x+1 = 3$

$-1 \quad -1$

$\frac{2x}{2} = \frac{2}{2}$

$x = \frac{2}{2} = 1$

9. $5^{3x-2} = 125$

$\log_5(5^{3x-2}) = \log_5(125)$

$125 = 5^{3}$

$3x-2 = 3$

$+2 \quad +2$

$\frac{3x}{3} = \frac{5}{3}$

$x = \frac{5}{3}$

10. $6^{x+2} = 216$

$\log_6(6^{x+2}) \quad \log_6(216)$

$216 = 6^{3}$

$x+2 = 3$

$-2 \quad -2$

$x = 1$

11. $7^{3x} = 343$

$\log_7(7^{3x}) = \log_7(343)$ $\quad 343 = 7^{3}$

$\frac{3x}{3} = \frac{3}{3}$

$x = 1$

12. $8^{x-1} = 4$

$\log_8(8^{x-1}) = \log_8(4)$ $\quad 4 = 8^{x} \rightarrow 2^2 = (2^3)^x = 2^{3x}$

$\log_2(2^2) = \log_2(2^{3x})$

$2 = 3x$

$\frac{2}{3} = x$

$x-1 = \frac{2}{3}$

$+1 \quad +1$ $\quad 1 = \frac{3}{3}$

$x = \frac{5}{3}$

4. $\log_5(x+4) = 2$

$$5^{\log_5(x+4)} = 5^2$$

$$x+4 = 25 \qquad x = 21$$

$$-4 \quad -4$$

5. $\log_{10}(x+5) = 3$

$$10^{\log_{10}(x+5)} = 10^3$$

$$x+5 = 1000 \qquad x = 995$$

$$-5 \quad -5$$

6. $\log_2(x-1) = 4$

$$2^{\log_2(x-1)} = 2^4$$

$$x = 17$$

$$x-1 = 16$$

$$+1 \quad +1$$

7. $\log_3(x-2) = 3$

$$3^{\log_3(x-2)} = 3^3$$

$$x-2 = 27 \qquad x = 29$$

$$+2 \quad +2$$

8. $\log_4(x-3) = 2$

$$4^{\log_4(x-3)} = 4^2 \qquad x = 19$$

$$x-3 = 16$$

$$+3 \quad +3$$

9. $\log_5(x-4) = 1$

$$5^{\log_5(x-4)} = 5^1 \qquad x = 9$$

$$x-4 = 5$$

$$+4 \quad +4$$

Answer: $(-5,-4)\cup(1,\infty)$

~~13~~ 1

$$\frac{x}{2(x+2)}-\frac{1}{x+2}\le\frac{12}{x(x+2)}$$

Zeros: $(x-6)(x+4)=0$

$x=6,-4$

VA: $2x(x+2)=0$

$x=0,-2$

$$\frac{x^2}{2x(x+2)}-\frac{2x}{2x(x+2)}\le\frac{24}{x(x+2)}$$

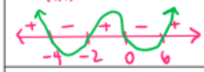
$$\frac{x^2-2x-24}{2x(x+2)}\le 0$$

$$\frac{(x-6)(x+4)}{2x(x+2)}\le 0$$

$f(-5)=\frac{(-)(-)}{(-)(-)}=+$ $\quad f(1)=\frac{(-)(+)}{(+)(+)}=-$

$f(-3)=\frac{(-)(+)}{(-)(-)}=-$ $\quad f(7)=\frac{(+)(+)}{(+)(+)}=+$

$f(-1)=\frac{(-)(+)}{(-)(+)}=+$



$[-4,-2)\cup(0,6]$

Answer: $[-4,-2)\cup(0,6]$

~~14~~ 2

$(2x^2+6x)+(-x-3)$

$2x(x+3)-(x+3)$

$$\frac{x+1}{2x^2+5x-3}\ge\frac{x}{4x^2-1}$$

Zeros: $x^2+1=0$

none

VA: $2x+1=0$

$x=-\frac{1}{2}$

$2x-1=0$

$x=\frac{1}{2}$

$x+3=0$

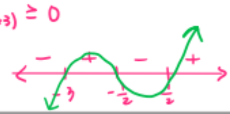
$x=-3$

$$\frac{x+1}{(2x-1)(x+3)}\ge\frac{x}{(2x+1)(2x-1)}$$

$$\frac{2x^2+3x+1}{(2x+1)(2x-1)(x+3)}\ge\frac{x^2+3x}{(2x+1)(2x-1)(x+3)}$$

$$\frac{x^2+1}{(2x+1)(2x-1)(x+3)}\ge 0$$

$(-3,-\frac{1}{2})\cup(\frac{1}{2},\infty)$



$f(-4)=\frac{+}{(-)(-)(-)}=-$

$f(-1)=\frac{+}{(-)(-)(+)}=+$

$f(0)=\frac{+}{(+)(-)(+)}=-$

$f(1)=\frac{+}{(+)(+)(+)}=+$

Answer: $(4,19]$

~~11~~ 7

$$\frac{x^2-11x-12}{x+4}<0$$

Zeros: $(x-12)(x+1)=0$

$x=12,-1$

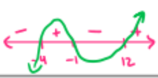
VA: $x+4=0$

$x=-4$

$$\frac{(x-12)(x+1)}{x+4}<0$$

$f(-5)=\frac{(-)(-)}{(-)}=-$ $\quad f(0)=\frac{(-)(+)}{+}=-$

$f(-2)=\frac{(-)(-)}{+}=+$ $\quad f(13)=\frac{(+)(+)}{+}=+$



$(-\infty,-4)\cup(-1,12)$

Answer: $\{-8\}\cup\{-7\}\cup[-5,\infty)$

~~9~~ 5

$$\left(\frac{x+6}{x+6}\right)-\frac{7}{x+5}\le-\frac{8}{x+6}\left(\frac{x+5}{x+5}\right)$$

Zeros: $x-2=0$

$x=2$

VA: $(x+6)(x+5)=0$

$x=-6,-5$

$$-\frac{7x+42}{(x+6)(x+5)}\le-\frac{8x+40}{(x+6)(x+5)}$$

$$\frac{-7x-42}{(x+6)(x+5)}+\frac{8x+40}{(x+6)(x+5)}\le 0$$

$$\frac{x-2}{(x+6)(x+5)}\le 0$$

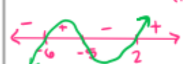
$$\frac{x-2}{(x+6)(x+5)}\le 0$$

$f(-7)=\frac{-}{(-)(-)}=-$

$f(-5.5)=\frac{-}{(+)(-)}=+$

$f(0)=\frac{-}{(+)(+)}=-$

$f(3)=\frac{+}{(+)(+)}=+$



$(-\infty,-6)\cup(-5,2]$

Answer: $[-5,5)\cup(6,\infty)$

~~8~~ 4

$$(x+8)^2(x+5)(x+7)^2\ge 0$$

$x=-8,-5,-7$

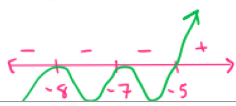
$f(-9)=(+)(-)(+)=-$

$f(-7.5)=(+)(-)(+)=-$

$f(-6)=(+)(-)(+)=-$

$f(0)=(+)(+)(+)=+$

$\{-8\}\cup\{-7\}\cup[-5,\infty)$



Answer: $\left(-3,-\frac{1}{2}\right)\cup\left(\frac{1}{2},\infty\right)$

~~7~~ 3

$$\left(\frac{x-6}{x-6}\right)-\frac{10}{x-5}\ge-\frac{11}{x-6}\left(\frac{x-5}{x-5}\right)$$

Zeros: $x+5=0$

$x=-5$

VA: $(x-6)(x-5)=0$

$x=6,5$

$$-\frac{10x-60}{(x-6)(x-5)}\ge-\frac{11x-55}{(x-6)(x-5)}$$

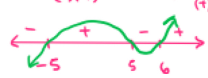
$$\frac{-10x+60}{(x-6)(x-5)}+\frac{11x-55}{(x-6)(x-5)}\ge 0$$

$$\frac{x+5}{(x-6)(x-5)}\ge 0$$

$f(-6)=\frac{-}{(-)(-)}=-$

$f(0)=\frac{+}{(-)(-)}=+$

$f(5.5)=\frac{+}{(-)(+)}=-$ $\quad f(7)=\frac{+}{(+)(+)}=+$



$[-5,5)\cup(6,\infty)$

Answer: $(-\infty,-6)\cup(-5,2]$

~~10~~ 6

$$\frac{2x+7}{x-4}\ge 3\left(\frac{x-4}{x-4}\right)$$

Zeros: $-x+19=0$

$x=19$

VA: $x-4=0$

$x=4$

$$\frac{2x+7}{x-4}\ge\frac{3x-12}{x-4}$$

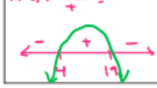
$$\frac{2x+7-3x+12}{x-4}\ge 0$$

$$\frac{-x+19}{x-4}\ge 0$$

$f(0)=\frac{+}{-}=-$

$f(5)=\frac{+}{+}=+$

$f(20)=\frac{-}{+}=-$



$(4,19]$

Answer: $(-\infty,-4)\cup(-1,12)$

~~12~~ 8

$$\frac{1}{x+5}+\frac{5}{x-1}>0$$

Zeros: $x+4=0$

$x=-4$

VA: $(x+5)(x-1)=0$

$x=-5,1$

$$\frac{x-1}{(x+5)(x-1)}+\frac{5x+25}{(x+5)(x-1)}>0$$

$$\frac{6x+24}{(x+5)(x-1)}>0$$

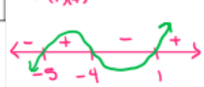
$$\frac{6(x+4)}{(x+5)(x-1)}>0$$

$f(-6)=\frac{-}{(-)(-)}=-$

$f(-4.5)=\frac{-}{(+)(-)}=+$

$f(0)=\frac{+}{(+)(-)}=-$

$f(2)=\frac{+}{(+)(+)}=+$



$(-5,4)\cup(1,\infty)$

- You learned that quantities that grow or decay exponentially increase or decrease at a constant percent rate.
- Some quantities have a constant doubling time or half-life.
- When the doubling time, d, or half-life, h, is known, the relationship between the initial amount, A_0, and the amount A after time t can be modelled by these equations:

Exponential Growth

- Doubling time is the time it takes for a population to double in size.

$$A = A_0(2)^{\frac{t}{d}}$$

Where: A = final quantity
A_0 = initial amount
2 = indicates doubling
t = time
d = doubling time

Exponential Decay

- Half-Life is the time it takes for a quantity to decay to half the original amount.

$$A = A_0(0.5)^{\frac{t}{h}}$$

Where: A = final quantity
A_0 = initial amount
0.5 = indicates half-life
t = time
h = half-life

PRACTICE:

1. Caffeine has a half-life of approximately 5 h. Suppose you drink a cup of coffee that contains 200 mg of caffeine. How long will it take until there is less than 10 mg of caffeine left in your bloodstream? Give your answer to 1 decimal place.

$h=5$ $A_0 = 200\text{mg}$ $t = ?$ $A = 10\text{mg}$

$$10 = 200(0.5)^{\frac{t}{5}}$$

$$\frac{1}{20} = 0.5^{t/5}$$

$$\log\left(\frac{1}{20}\right) = \log\left(0.5^{t/5}\right)$$

$$\log\left(\frac{1}{20}\right) = \frac{t}{5}\log(0.5)$$

$$\frac{5\log\left(\frac{1}{20}\right)}{\log\left(\frac{1}{2}\right)} = t$$

$t = 21.6$ hours

2. Tritium, a radioactive gas that builds up in CANDU nuclear reactors, is collected, stored in pressurized gas cylinders, and sold to research laboratories. Tritium decays into helium over time. Its half-life is about 12.3 years. $h = 12.3$

a) Write an equation that gives the mass of tritium remaining in a cylinder that originally contained 500 g of tritium. $A(t) = 500(0.5)^{t/12.3}$

b) Estimate the time it takes until less than 5 g of tritium is present.

$5 = 500(0.5)^{t/12.3}$ $\quad \frac{1}{100} = 0.5^{t/12.3}$ $\quad \log\left(\frac{1}{100}\right) = \frac{t}{12.3}(\log(0.5))$

$[12.3\log(\frac{1}{100})]/\log(0.5) = t = 81.7$ years

$t = ?$ 3. A colony of bacteria doubles in size every 20 min. How long will it take for a colony of 20 bacteria to grow to a population of 10 000?

$d = 20$ $\quad A_0 = 20$ $\quad A = 10{,}000$ $\quad 10{,}000 = 20(2)^{\frac{t}{20}}$ $\quad 500 = 2^{\frac{t}{20}}$ $\quad \log(500) = \frac{t}{20}\log(2)$

$t = 179.3$ mins

4. An archaeologist uses radiocarbon dating to determine the age of a Viking ship. Suppose that a
$h = 5730$ sample that originally contained 100 mg of Carbon-14 now contains 85 mg of Carbon-14.What is the age of the ship to the nearest hundred years?

$85 = 100(0.5)^{t/5730}$ $\quad \log(0.85) = \frac{t}{5730}\log(0.5)$ $\quad$ 1300 years

5. A bacteria culture starts with 6 500 bacteria. After 2.5 hours, there are 208 000 bacteria present. What is the length of the doubling period?

$208{,}000 = 6500(2)^{2.5/d}$ $\quad \log(32) = \frac{2.5}{d}\log(2)$ $\quad d = \frac{2.5 \cdot \log(2)}{\log(32)} = 0.5$ hours

6. A bacteria culture doubles every 0.25 hours. At time 1.25 hours, there are 40 000 bacteria present. How many bacteria were present initially?

$40\,000 = A_0(2)^{\frac{1.25}{0.25}}$ $\quad 40000 = A_0(2^5)$ $\quad A_0 = 1250$

7. A sodium isotope, Na^{24}, has a half-life of 15 hours. Determine the amount of sodium that remains from a 4 g sample after:

a) 45 hours 0.5g $\quad A = 4(0.5)^{\frac{45}{15}}$

b) 100 hours 0.04g $\quad A = 4(0.5)^{\frac{100}{15}}$

c) 5 days = 120 hours 0.02g $\quad A = 4(0.5)^{\frac{120}{15}}$

8. The 50 cent Bluenose is one of Canada's most famous postage stamps. In 1930 it could be bought at the post office for \$0.50. In 2000, a stamp in excellent condition was sold at an auction for \$512. Determine the doubling time for the stamp's value. $2000 - 1930 = 70$ years

$512 = 0.5(2)^{\frac{70}{d}}$ $\quad \log(1024) = \frac{70}{d}\log(2)$ $\quad \frac{70 \cdot \log(2)}{\log(1024)} = d = 7$

9. Strontium-90 has a half-life of 25 years. How long would it take for 40 mg of it to decay to:

a) 20 mg 25 years

$20 = 40(0.5)^{\frac{t}{25}}$ $\quad \log(0.5) = \frac{t}{25}\log(0.5)$ $\quad 25 = t$

b) 1.25 mg 125 years

$1.25 = 40(0.5)^{t/25}$ $\quad \log(0.03125) = \frac{t}{25}\log(0.5$