Transformations of Sinusoidal Functions

- You can apply the same transformation rules to sinusoidal functions of the form $y = a \sin b(\theta - c) + d$ or $y = a \cos b(\theta - c) + d$.
  - A vertical stretch by a factor of $|a|$ changes the amplitude to $|a|$.

    $y = a \sin \theta$ $\qquad$ $y = a \cos \theta$

    If $a < 0$, the function is reflected through the horizontal mid-line of the function.
  - A horizontal stretch by a factor of $\frac{1}{|b|}$ changes the period to $\frac{360°}{|b|}$ or $\frac{2\pi}{|b|}$ radians.

    $y = \sin(b\theta)$ $\qquad$ $y = \cos(b\theta)$

    If $b < 0$, the function is reflected in the y-axis.
  - For sinusoidal functions, a horizontal translation is called the *phase shift*.

    $y = \sin(\theta - c)$ $\qquad$ $y = \cos(\theta - c)$

    If $c > 0$, the function shifts c units to the right.
    If $c < 0$, the function shifts c units to the left.
  - The *vertical displacement* is a vertical translation.

    $y = \sin \theta + d$ $\qquad$ $y = \cos \theta + d$
  - If $d > 0$, the function shifts d units up.
  - If $d < 0$, the function shifts d units down.

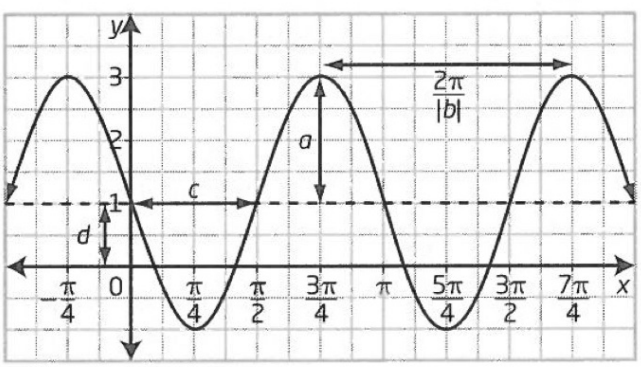
    $$d = \frac{\text{maximum value} + \text{minimum value}}{2}$$
  - The *sinusoidal axis* is defined by the line $y = d$. It represents the mid-line of the function.

- Apply transformations of sinusoidal functions in the same order as for any other functions:
  i) horizontal stretches and reflections, $\frac{1}{|b|}$
  ii) vertical stretches and reflections, $|a|$
  iii) translations, c and d

- The domain of a sinusoidal function is not affected by transformations.

  The range of a sinusoidal function, normally $\{y \mid -1 \leq y \leq 1, y \in \mathrm{R}\}$, is affected by changes to the amplitude and vertical displacement.

  Consider the graph of $y = 2 \sin 2\left(x - \frac{\pi}{2}\right) + 1$.



$a = 2$, so the amplitude is 2

$b = 2$, so the period is $\frac{2\pi}{2}$, or π

$c = \frac{\pi}{2}$, so the graph is shifted $\frac{\pi}{2}$ units right

$d = 1$, so the graph is shifted 1 unit up

domain: $\{x \mid x \in \mathrm{R}\}$

range: $\{y \mid -1 \leq y \leq 3, y \in \mathrm{R}\}$

Identify period, amplitude, and shift of sine and cosine functions from an equation
Worksheet

Name: _____

Date: _____ Period: _____

1. Find the period of the function defined below.

$y = \sin\left(\frac{5\pi}{4}x\right)$

2. Find the period of the function defined below.

$y = \cos\left(\frac{3\pi}{2}x\right)$

3. Find the amplitude of the function defined below.

$y = -8 \cdot \sin\left(\frac{\pi}{3}x\right)$

4. Find the amplitude of the function defined below.

$y = 4 \cdot \cos\left(\frac{7\pi}{6}x\right)$

5. Find the phase shift and vertical shift of the function defined below.

$y = -10 \cdot \sin\left(x + \frac{\pi}{6}\right) - \frac{11}{8}$

6. Find the phase shift and vertical shift of the function defined below.

$y = 11 \cdot \cos\left(x + \frac{\pi}{4}\right) + \frac{1}{2}$

7. Find the phase shift and vertical shift of the function defined below.

$y = 2 \cdot \sin\left(x + \frac{5\pi}{3}\right) - \frac{4}{9}$

8. Find the phase shift and vertical shift of the function defined below.

$y = \frac{5}{2} \cdot \cos\left(4\left(x - 4\right)\right) - 9$

9. Find the frequency of the function defined below.

$y = \sin\left(\frac{11\pi}{6}x\right)$

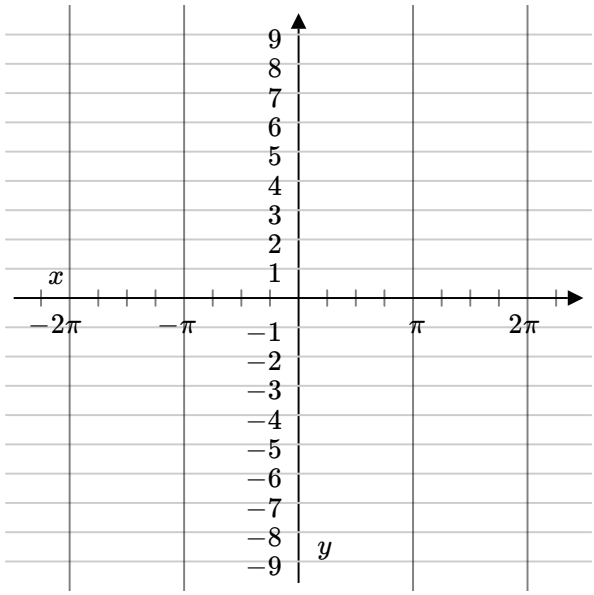
10. Find the frequency of the function defined below.

$y = \cos\left(\frac{5\pi}{4}x\right)$

Finding Key Points to Graph Sinusoidal Functions

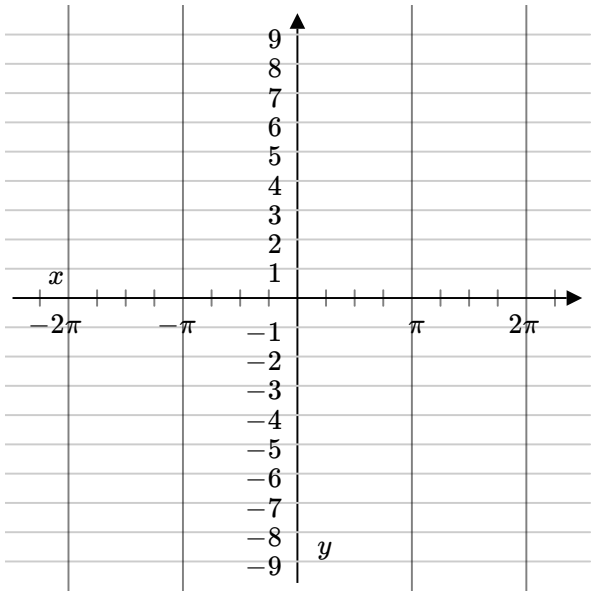
1. Graph the sine function below.

$y = \sin\left(x + \frac{3\pi}{4}\right) - 5$



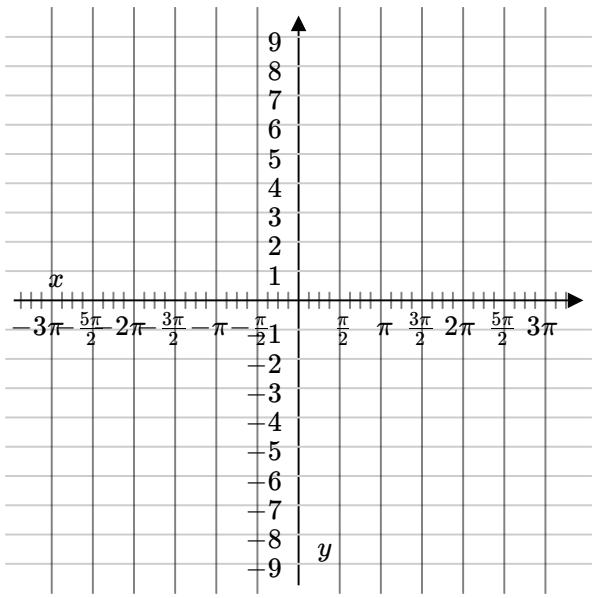
2. Graph the sine function below.

$y = \sin(x + \pi) - 3$



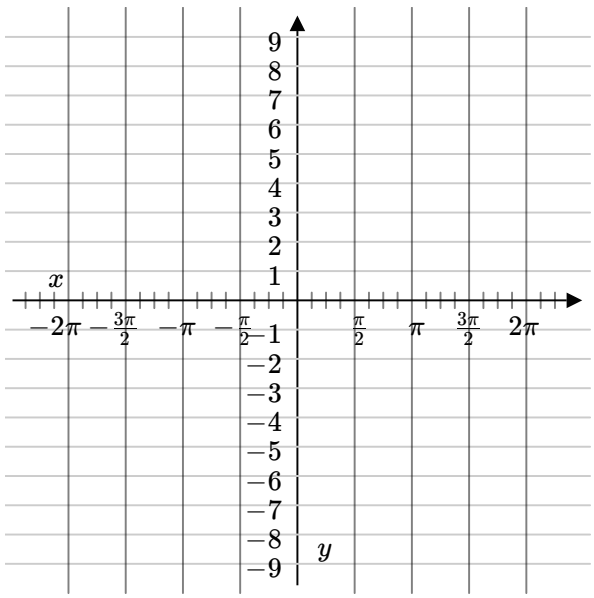
3. Graph the sine function below.

$y = 4 \cdot \sin\left(\frac{4}{5}x\right)$



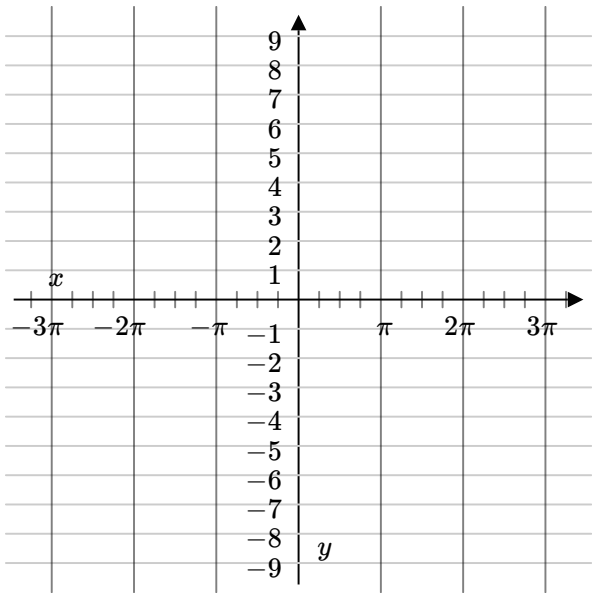
4. Graph the sine function below.

$y = 3 \cdot \sin(-4x)$



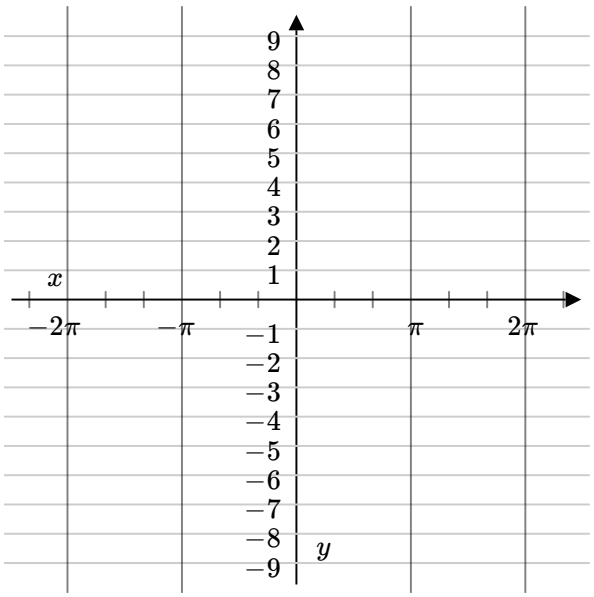
5. Graph the sine function below.

$y = \sin\left(-\frac{2}{3}\left(x + \frac{3\pi}{4}\right)\right)$



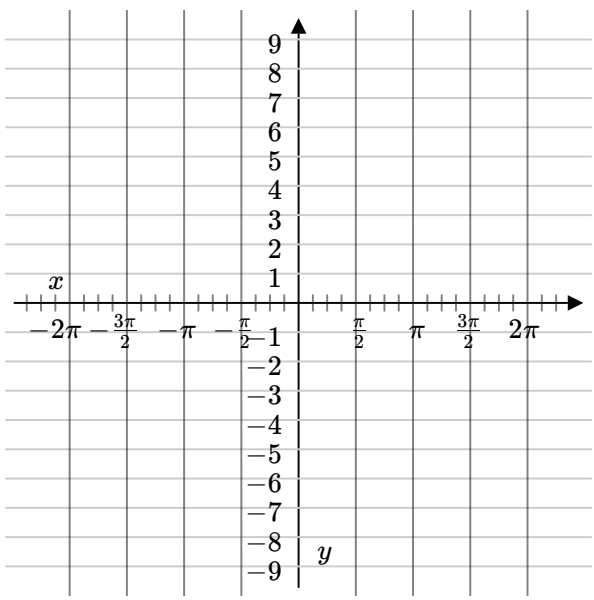
6. Graph the sine function below.

$y = \sin\left(\frac{3}{2}\left(x - \frac{\pi}{3}\right)\right)$



7. Graph the sine function below.

$y = 4 \cdot \sin\left(\frac{4}{3}\left(x + \frac{\pi}{2}\right)\right) + 5$



8. Graph the sine function below.

$y = -4 \cdot \sin\left(\frac{4}{3}\left(x + \frac{\pi}{4}\right)\right) + 1$